AERO
MATHEMATICS CURRICULUM FRAMEWORK
K-8 STANDARDS AND PERFORMANCE INDICATORS

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Teaching Mathematics for the 21st Century

We need citizens who can problem solve and think critically to compete in an ever-changing technological and global society. We must produce students who are capable of becoming life-long learners and successful citizens in a global market place. Therefore, students must develop a deep understanding of mathematical concepts and possess a strong foundation of number sense in order to become proficient in mathematics.

Every teacher of mathematics has an individual goal to provide students with the knowledge and understanding of the mathematics necessary to function in a world very dependent upon the application of mathematics. Instructionally, this goal translates into three components: conceptual understanding; procedural fluency; problem solving

Conceptual understanding consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts. Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. Knowledge learned with understanding provides a foundation for remembering or reconstructing mathematical facts and methods, for solving new and unfamiliar problems, and for generating new knowledge.

Procedural fluency is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). Although the word procedural may imply an arithmetic procedure to some, it also refers to being fluent with procedures from other branches of mathematics, such as measuring the size of an angle using a protractor. The use of calculators need not threaten the development of students' computational skills. On the contrary, calculators can enhance both understanding and computing if used properly and effectively. Accuracy and efficiency with procedures are important, but they should be developed through understanding. When students learn procedures through understanding, they are more likely to remember the procedures and less likely to make common computational errors.

Problem solving is the ability to formulate, represent, and solve mathematical problems.
Problems generally fall into three types:

- one-step problems
- multi-step problems
- process problems

Most problems that students will encounter in the real world are multi-step or process problems. Solution of these problems involves the integration of conceptual understanding and procedural knowledge. Students need to have a broad range of strategies upon which to draw. Selection of a strategy for finding the solution to a problem is often the most difficult part of the solution. Therefore, mathematics instruction must include the teaching of many strategies to empower all students to become successful problem solvers. A concept or procedure in itself is not useful in problem solving unless one recognizes when and where to use it as well as when and where it does not apply. Therefore, students need to be able to have a general understanding of how to analyze a problem and how to choose the most useful strategy for solving the problem.

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Individually, each of these components (conceptual understanding, procedural fluency, and problem solving) is necessary but not sufficient for a student to be mathematically proficient. They are not, however, independent of each other. They are integrally related, need to be taught simultaneously, and should be a component of every lesson. In this document conceptual understanding, procedural fluency, and problem solving are represented as process strands and content strands. These strands help to define what students should know and be able to do as a result of their engagement in the study of mathematics.

Process Strands: The process strands (Problem Solving, Reasoning and Proof, Communication, and Connections/ Representation) highlight ways of acquiring and using content knowledge. These process strands help to give meaning to mathematics and help students to see mathematics as a discipline rather than a set of isolated skills. Student engagement in mathematical content is accomplished through these process strands. Students will gain a better understanding of mathematics and have longer retention of mathematical knowledge as they solve problems, reason mathematically, prove mathematical relationships, participate in mathematical discourse, make mathematical connections, and model and represent mathematical ideas in a variety of ways.

The Content of Mathematics:

Mathematics is a tool we use to understand and interpret our world. In our increasingly technological economy, those who can understand and apply mathematics have significantly enhanced opportunities to achieve success in continuing education and in life. The key to opening the door to these opportunities is a deep understanding of important mathematical concepts and procedures. The mathematical content must be coherent and vertically articulated across the grades.

The AERO Mathematics Curriculum Framework connects the Process and Content Standards. The Process Standards describe the process in which students should learn mathematics and engage in mathematical thinking. The Content Standards outline the big mathematical ideas that all students should know and be able to do at each grade level. The relationship between the Process and Content Standards is critical. It is the combination of these two standards that will give students mathematical power. Neither will develop mathematically proficient students when used in isolation. Teachers are expected to use instructional practices that provide opportunities for students to experience both Process and Content Standards on a regular basis.

NOTE: There is a difference between the intent of the AERO Mathematics Curriculum Framework and the Common Core. The Common Core was developed from the standpoint of instruction and not only includes the math required of all students but also describes additional mathematics that students should learn to pursue careers and majors in science, technology, engineering and mathematics **(STEM) fields indicated by a + sign**. The AERO Mathematics Framework was developed as a document to inform assessments. The AERO Mathematics Curriculum Framework has been adopted from the Common Core.

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Depth-of-Knowledge (DOK) Levels for Mathematics

Each indicator for the AERO Mathematics Curriculum Framework has been assigned a Depth of Knowledge (DOK) level based on the work of Norman L. Webb, Wisconsin Center for Educational Research ("Depth-of-Knowledge Levels for Four Content Areas," March 28, 2002), DOK levels measure the degree to which the knowledge elicited from students on assessments are as complex as what students are expected to know and do as stated in the performance indicators. According to Webb, "interpreting and assigning depth-of-knowledge levels to both objectives within standards and assessment items is an essential requirement of alignment analysis. Instruction, assignments, and classroom assessment must incorporate the same expectations. DOK levels for an indicator must mirror the DOK level for the assessment.

DOK levels help administrators, teachers, and parents understand the intent of the indicators, in terms of the complexity of what students are expected to know and do. Indicators vary in terms of complexity. Some expect students to reproduce a fact or complete a sequence of steps, while others expect students to reason, extend their thinking, synthesize information from multiple sources, and produce significant work over time. Teachers must know what level of complexity is required by an indicator in order to ensure that students have received prior instruction or have had an opportunity to learn content at the level students will be expected to demonstrate or perform. Assessment items must be created to ensure that what is elicited from students on the assessment is as demanding cognitively as what students are expected to know and do as stated in the indicators.

Four levels of Depth of Knowledge (DOK) are used in the AERO Mathematics Curriculum Framework.

DOK 1 (Recall) includes the recall of information such as fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

DOK 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

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DOK 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections-relate ideas within the content area or among content areas-and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level.

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Process Standards

Standard 1 (Problem Solving),
Standard 2(Reasoning and Proof),
Standard 3(Communication and Reasoning)
Standard 4 (Connections)

Enduring Understandings:

Mathematics can be used to solve problems outside of the mathematics classroom.
Mathematics is built on reason and always makes sense.
Reasoning allows us to make conjectures and to prove conjectures.
Classifying helps us build networks of mathematical ideas.
Precise language helps us express mathematical ideas and receive them.

Essential Questions:

Is your plan working?
Do you need to reconsider what you are doing?
How are solving and proving different?
How are showing and explaining different?
How do you know when you have proven something?
How do you develop a convincing argument?
How do you make sense of different strategies?
How do you determine the strengths and weaknesses of different strategies?
How do you determine similarities and differences of different strategies?
Why do we classify numbers?
Why do we classify geometric objects?

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1.0 Problem Solving

Problem solving means engaging in a task for which the solution process is not known in advance. Good problem solvers have developed a "mathematical disposition" which allows them to analyze situations in mathematical terms. They have developed a range of strategies for developing a solution to a problem, have learned to monitor and adjust the strategies they choose to use in the process of solving a specific problem, and can compare and contrast solutions and problems.

Students will develop their ability to SOLVE PROBLEMS by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply strategies to solve a wide variety of problems; and to integrate mathematical reasoning, communication, and connections.. All students in grades K–8 will be able to:

- Develop and apply strategies to solve problems.
- Use mathematical notation and language to explain and defend their thinking
- Make and test conjectures in a variety of mathematical situations.
- Evaluate the reasonableness of the solution in the context of the original situation.

Standard	**1.0 Students will apply a wide variety of mathematical concepts, processes, and skills to solve a broad range of problems in various content areas and everyday situations.**								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Indicator	**Students will be able to**								
Problem Solving Examine	Identify questions to be answered when solving a problem.			Analyze a problem to determine the question(s) to be answered.			Extract and organize mathematical information for a given purpose, such as making conjectures or drawing conclusions		
	Identify what is known and unknown in a problem and recognize when information is missing.			Identify necessary and extraneous information			Identify necessary and extraneous information		

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Level	K	1	2	3	4	5	6	7	8
Indicator	**Students will be able to:**								
Problem Solving Plan	NA			Determine an efficient strategy, verify, interpret, and evaluate the results with respect to the original problem			Determine an efficient strategy, verify, interpret, and evaluate the results with respect to the original problem		
Problem Solving Explore	Try more than one strategy when the first strategy proves to be unproductive			Try more than one strategy when the first strategy proves to be unproductive			Apply problem solving strategies until a solution is found or it is clear that no solution exists		
Problem Solving Solve	Solve problems, choosing from a variety of problem-solving strategies such as drawing pictures, manipulating objects, using numbers, or acting out the situation.			Select and use strategies and procedures to find solutions to problems .			.Identify relevant mathematical information in a problem situation and select and use the strategy to solve a problem.		
	NA			Interpret and solve a variety of mathematical problems by paraphrasing			Interpret and solve a variety of mathematical problems by paraphrasing		
	Check the reasonableness of a solution			Check the reasonableness of a solution			Check the reasonableness of a solution		
	Explain and verify results with respect to the original problem			Generalize and apply previous experiences and strategies to new problem solving situations			Generalize solutions and apply previous knowledge to new problem solving situations		

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Level	K	1	2	3	4	5	6	7	8
Indicator	**Students will be able to:**								
Appropriate Technology and Models	**NA**			Use technology, including calculators, to develop mathematical concepts			Apply technology as a tool in problem solving situations		

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2.0 Reasoning and Proof

Systematic reasoning is a defining feature of mathematics. Exploring, justifying, and using mathematical conjectures are common to all content areas and, with different levels of rigor, all grade levels. Students will develop their Reasoning and **Proof** ability by solving problems in which there is a need to investigate significant mathematical ideas in all content areas; to **justify** their thinking; to reinforce and extend their logical reasoning abilities; to reflect on and clarify their own thinking; to ask questions to extend their thinking; and to construct their own learning. All students in grades K–12 will be able to:

- Reinforce and extend their logical reasoning abilities
- Reflect on, clarify, and justify their thinking
- Ask questions to extend their thinking
- Use patterns and relationships to analyze mathematical situations
- Determine relevant, irrelevant, and/or sufficient information to solve mathematical problems

Standard	**2.0 Students will apply mathematical reasoning skills to investigate, evaluate, justify, and connect approaches and solutions to situations in mathematics and in other disciplines.**								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Indicator	**Students will be able to:**								
Reasoning and Proof:	Draw logical conclusions about mathematical problems			Draw logical conclusions about mathematical problems			Draw logical conclusions about mathematical problems		
	NA			Follow a logical argument and judge its validity			Recognize and apply deductive and inductive reasoning		
	Discuss the steps used to solve a mathematical problem			Review and refine the assumptions and steps used to derive conclusions in mathematical arguments			Review and refine the assumptions and steps used to derive conclusions in mathematical arguments		
	Justify and explain the solutions to problems using physical models			Justify and explain the solutions to problems using manipulatives and physical models			Justify answers and the steps taken to solve problems with and without manipulatives and physical models		

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3.0 Communication and Representation

As students are asked to communicate orally or in writing about the mathematics they are studying, they gain insights into their own thinking. In order to communicate their thinking to others, they naturally reflect on their learning and organize and consolidate their thinking about mathematics. Students should be encouraged and expected to increase their ability to express themselves clearly and coherently over time. In particular, the ability to express thoughts and describe solutions in writing should be a major focus of the mathematics curriculum.

Representations are necessary to students' understanding of mathematical concepts and relationships. They allow students to communicate mathematical approaches, arguments, and understandings to themselves and others. Appropriate representations allow students to recognize connections among related concepts, and lead to efficient methods of solving problems. It is important to encourage students to represent their mathematical ideas inways that make sense to them, even if those representations are not conventional. At the same time, students should learn conventional forms of representation in ways that facilitate their learning of mathematics and their communication with others about mathematical ideas.

Students will develop their mathematical Communication ability by solving problems in which there is a need to obtain information from the real world through reading, listening and observing; to translate this information into mathematical language and symbols; to process this information mathematically; and to present results in written, oral, and visual formats. All students in grades K–12 will be able to:

- Translate information into mathematical language and symbols
- Process information mathematically
- Present results in written, oral, and visual formats
- Discuss and exchange ideas about mathematics as a part of learning
- Read a variety of fiction and nonfiction texts to learn about mathematics
- Use representations to model, communicate and explain problems
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems

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Standard	3.0 Students will accurately and clearly present and justify mathematical ideas in diverse formats.								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Indicator	**Students will be able to**								
Communication	Use inquiry techniques to solve mathematical problems			Use inquiry techniques to solve mathematical problems			Use formulas, algorithms, inquiry, and other techniques to solve mathematical problems		
	Use physical materials, models, pictures, or writing to represent and communicate mathematical ideas			Use a variety of methods to represent and communicate mathematical ideas through oral, verbal, and written formats			Evaluate written and oral presentations in mathematics		
	Identify and translate key words and phrases that imply mathematical operations			Identify and translate key words and phrases that imply mathematical operations			Identify and translate key words and phrases that imply mathematical operations		
	NA			**NA**			Model and explain mathematical relationships using oral, written, graphic, and algebraic methods		
	Explain what they did to solve a problem.			Use informal and mathematical language to explain why certain strategies or procedures were used to find a solution.			Use appropriate representations, symbols, and informal and formal mathematical language to communicate mathematical thinking coherently and clearly.		

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4.0 Connections

Mathematics is an integrated field of study, even though it is often studied in separate areas or topics. Viewing mathematics as a whole helps students learn that mathematics is not a set of isolated skills and arbitrary rules. Focusing on mathematics in context and establishing mathematical connections makes it easier to apply mathematical knowledge and makes it less likely that students will forget or misapply important mathematical skills and rules.

Students will develop mathematical Connections by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways. All students in grades K–12 will be able to:

- Link new concepts to prior knowledge
- Identify relationships between content strands
- Allow the flexibility to approach problems in a variety of ways within and beyond the field of mathematics
- Recognize and apply mathematics in contexts outside of mathematics

Standard	4.0 Students will develop the ability to use connections among mathematical ideas to build on one another when solving real-world problems and to interconnect ideas to produce an integrated coherent whole.								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Indicators	**Students will be able to**								
Connections	Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as rhythm in music and motion in science			Use mathematical ideas from one area of mathematics to explain an idea from another area of mathematics			Use mathematical ideas from one area of mathematics to explain an idea from another area of mathematics		
	NA			Use physical models to explain the relationship between concepts and procedures			Use manipulatives and physical models to explain the relationships between concepts and procedures		

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Level	K	1	2	3	4	5	6	7	8
Indicators	**Students will be able to**								
Connections	**NA**			**NA**			Use the connections among mathematical topics to develop multiple approaches to problems		
	NA			Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as rhythm in music and motion in science			Apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as rhythm in music and motion in science		
	Identify mathematics used in everyday life			Identify, explain, and use mathematics in everyday life			Identify, explain, and apply mathematics in everyday life		

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5.0 NUMBERS AND OPERATIONS

Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts. Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Enduring Understandings:

Numbers can be represented in multiple ways.
The same operations can be applied in problem situations that seem quite different from another.
Being able to compute fluently means making smart choices about which tools to use and when to use them.
Knowing the reasonableness of an answer comes from using good number sense and estimation strategies.

Essential Questions:

1. What makes an estimate reasonable?
2. What makes an answer exact?
3. What makes a strategy both effective and efficient?
4. What makes a solution optimal?

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Standard	**5.0. Students will understand and apply numbers, ways of representing numbers, relationships among numbers, and number systems**.								
Benchmark	**5.1 Numbers and Number Sense** **Students will understand and demonstrate a sense of what numbers mean and how they are used. Students will be able to**:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Counting	Count forward by 1's and tens to 100 and backward from 100 with and without objects DOK 1	Count forward by 1's to 120, with and without objects and starting with any number less than 120, and count by two's to at least 100 DOK 1	Count by fives, tens and hundreds starting at any number from 1 to 999 DOK 1	Count by hundreds and thousands starting at any number from 1 to 9,999 DOK 1	Count by thousands and ten thousands starting at any number from 1 to 99,999 DOK 1	Count by thousands, ten thousands, and hundred thousands, starting at any number from 1 to 999,999 DOK 1	**NA**	**NA**	**NA**
Counting Sets	Count how many objects are in a set of up to 20 objects and count out a specific number of objects (up to 20) from a larger set. DOK 1	Group and count objects by twos, tens, and fives to 100. DOK 1	Name the number that is 1 more than or 10 more than any number from 0 through 999 and 1 less than or 10 less than any number from 10 through 1000. DOK 1	Name the number that is 10 more than or 100 more than any number from 0 through 9,999 and 10 less than or 100 less than any number from 100 through 10,000. DOK 1	Name the number that is 100 more than or 1000 more than any number from 0 through 99,999 and 100 less than or 1000 less than any number from 1000 through 100,000. DOK 1	**NA**	**NA**	**NA**	**NA**

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Level	K	1	2	3	4	5	6	7	8
Reading and Writing Numbers	Identify, write, and read aloud numbers from 0 to at least 31 DOK 1 .	Identify, read aloud and write numbers to 100 DOK 1	Identify, read aloud and write numbers to 1000. DOK 1	Identify, read aloud and write numbers to 10,000. DOK 1	Read and write numbers to at least 100,000 DOK 1	Read and write numbers to at least 1,000,000 DOK 1	**NA**	**NA**	**NA**
Ordering and Comparing (Whole Numbers)	Compare two sets of up to 20 objects each and explain why the number of objects in one set is equal to, greater than, or less than the number of objects in the other set. DOK 2	Write, compare, and order numbers to at least 100 using the words equal to, greater than, less than, greatest, and least and recording the results of comparisons with the symbols >, =, and <. When appropriate DOK 1	Compare and order numbers from 0 to at least 1,000 using the words equal to, greater than, less than, greatest, or least and recording the results of comparisons with the symbols >, =, and <. when appropriate. DOK 1	Compare and order numbers from 0 to at least 10,000 using the words equal to, greater than, less than, greatest, or least and recording the results of comparisons with the symbols >, =, and <. when appropriate. DOK 1	Compare and order numbers from 0 to at 100,000 using the words equal to, greater than, less than, greatest, or least and recording the results of comparisons with the symbols >, =, and <. when appropriate DOK 1	Compare and order numbers from 0 to at 1,000,000 using the words equal to, greater than, less than, greatest, or least and recording the results of comparisons with the symbols >, =, and <. when appropriate DOK 1	NA	NA	NA

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Level	K	1	2	3	4	5	6	7	8
Ordering and Comparing (Numbers)	Use a number line or chart to locate and identify the numbers (from 1 to 100) coming before/after a given number and between 2 given numbers DOK 1	Use a number line or chart, locate, compare, and order whole numbers less than 100 and identify the numbers coming before/after a given number and between 2 given numbers DOK 1	Use words, number lines, and models to compare, and order whole numbers through 999 DOK 2	Use symbols (i.e., <, =, >) and models to compare and order whole numbers through 9,999 DOK 2	Use symbols (i.e., <, =, >) and models to compare and order whole numbers through 99,999 DOK 2	Read, write, compare, and order all whole numbers, fractions, mixed numbers and decimals using multiple strategies (e.g. symbols, manipulatives, number line, and place value concepts) DOK 2	Read, write, compare, and order integers using multiple strategies (e.g., symbols, manipulatives, number line). DOK 1	Compare, order, and differentiate among integers, decimals, fractions, and irrational numbers using multiple representations (e.g., symbols, manipulatives, graphing on a number line). DOK 2	Analyze, describe and compare the characteristics of rational and irrational numbers. DOK 2
Place Value	Identify place value of each digit utilizing standard and expanded form through 20. DOK 1	Construct models and identify place value of each digit utilizing standard and expanded form through 99. DOK 2	Construct models and identify place value of each digit utilizing standard and expanded form through 999. DOK 2	Model and identify place value of each digit utilizing standard and expanded form through 9999. DOK 1	Identify and interpret the place value for each digit in numbers through 99,999 DOK 1	Identify and use place value positions of whole numbers and decimals to hundredths DOK 1	Identify and use place value positions of whole numbers and decimals to thousandths DOK 1	Write, identify, and use (standard and expanded form) powers of 10 from 10^{-3} through 10^{6} DOK 2	Represent numbers using scientific notation in mathematical and practical situations. DOK 2
Exponents	NA	NA	NA	NA	NA	NA	NA	Explain the relationship between standard form and scientific notation. DOK 2	Recognize and appropriately use exponential and scientific notation. DOK 1

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Level	K	1	2	3	4	5	6	7	8
Ordering and Comparing (Ordinal Numbers)	Use the ordinal numerals 1st through 10th to discuss positions in ordered lists DOK 1	Use ordinal numbers 1st – 20th to identify position in a sequence DOK 1	Use ordinal numbers through 31st as they relate to the calendar DOK 1	NA	NA	NA	NA	NA	NA
Fractions and Decimals	Identify and name halves and whole using concrete items DOK 1	Identify and name halves, thirds, and fourths as part of a whole and as part of a group using models DOK 1	Represent fractions that have denominators ranging from 2 to 12 using physical objects, pictures, numbers, and words, and translate among representations DOK 2	Use concrete models and pictorial representations to demonstrate the meaning of fractions (proper and improper) as parts of a whole, parts of a set, and division by whole numbers through twelfths DOK 2	Use models to connect and compare equivalent fractions and decimals. DOK 1	Determine decimal equivalents or approximations of common fractions (i.e., ¼, ½, ¾, and 1 whole) DOK 1 .	Determine decimal and percent equivalents including approximations for common fractions (i.e., ¼, ½, ¾, and 1 whole). DOK 1	Represent rational numbers as fractions, mixed numbers, decimals or percents and convert among various forms as appropriate DOK 2	Classify numbers as rational or irrational. DOK 1

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Level	K	1	2	3	4	5	6	7	8
Equivalent Fractions	NA	NA	Identify that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one. DOK 1	Identify, name and use equivalent fractions with denominators 2, 4 and 8. DOK 1	Write a fraction equivalent to a given fraction using common multiples. And simplify fractions using common factors. DOK 1	Relate equivalent fractions and decimals with and without models, including locations on a number line. DOK 1	Compute equivalent representations of fractions and decimals (i.e., halves, thirds, fourths, fifths, eighths, tenths, hundredths) DOK 1	Model and identify equivalent fractions including conversion of improper fractions to mixed numbers and vice versa. DOK 2	Use rational and irrational numbers to solve real-world and mathematical problems. DOK 2
Compare and order fractions and decimals	**NA**	**NA**	Place 0 and halves on the number line from 0 to 10 DOK 1	Compare and order fractions by using models, benchmarks (0, ½, 1), or common numerators or denominators DOK 2	Compare and order positive fractions (including positive mixed numbers) and decimals on the number line, in number sentences, and in lists DOK 2	Use models and drawings, and find common denominators to compare fractions with unlike denominators DOK 2	Compare positive fractions, decimals, and positive and negative integers using symbols (i.e., <, =, >) and number lines DOK 2	Compare and order combinations of **rational and irrational numbers** DOK 2	Represent and compare rational and irrational numbers symbolically and on a number line. DOK 2

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Level	K	1	2	3	4	5	6	7	8
Representing Decimals	NA	NA	NA	Use numbers, words, pictures, and physical objects to read, write, and represent decimal numbers (to the tenths) between 0 and 1, between 1 and 2, etc. DOK 1	Use numbers, words, pictures, and physical objects to read, write, and represent decimal numbers (to the hundredths) between 0 and 1, between 1 and 2, etc. DOK 1	Round, order, and compare, using symbols, decimals to the tenths, hundredths, and thousandths place DOK 1	Read, write, compare, and order groups of decimals DOK 1	NA	NA
Equivalency Decimals and Fractions	NA	NA	Distinguish the equivalency among decimals, fractions and percents (e.g., half- = 50%). DOK 2	Determine the equivalency among decimals, fractions, and percents (e.g., half = 50% and ¼ = 0.25 = 25%). DOK 2	Determine the equivalency among decimals, fractions, and percents (e.g., 49/100 = 0.49 = 49%). DOK 2	Determine the equivalency between and among fractions, decimals, and percents in contextual situations. DOK 2	Determine the equivalency between and among fractions, decimals, and percents in contextual situations. DOK 2	Express fractions as terminating or repeating decimals. DOK 1	NA
Counting Money	Identify and sort coins of the host country DOK 1	Find the value of any set of coins using one denomination of coins DOK 1	Determine the value of a given set of coins DOK 1	Determine possible combinations of coins and bills to equal given amounts DOK 2	Determine totals for monetary amounts in practical situations DOK 1	Determine totals, differences, and change due for monetary amounts in practical situations DOK 1	Compare and use unit cost in practical situations DOK 2	Calculate simple interest in monetary problems DOK 1	Calculate percents in monetary problems DOK 1

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Level	K	1	2	3	4	5	6	7	8
Money Notation	**NA**	**NA**	Use decimals to show money amounts DOK 1	Read, write and use money notation DOK 1	Use money notation to add and subtract given monetary amounts DOK 1	**NA**	**NA**	**NA**	**NA**
Integers	NA	NA	NA	NA	Illustrate the meaning of positive and negative integers using models, such as the number line or colored chips, and situations, such as elevation or temperature DOK 2	Describe real-world situations using positive and negative numbers DOK 1	Explain the meaning and relationship between absolute value and opposites. DOK 2	Develop, analyze, and apply models (including everyday contexts), strategies, and procedures to compute with integers, with an emphasis on negative integers. DOK 3	Simplify expressions involving operations on integers, grouping symbols, and whole number exponents using order of operations DOK 1
Ratios	**NA**	**NA**	**NA**	Describe relationships between quantities using ratios. DOK 1	Make comparisons and describe quantitative relationships using ratios. DOK 2	Represent ratios and proportions and solve problems using models and pictures. DOK 2	Compare quantities and solve problems using ratios, rates and percents. DOK 2	Solve problems using ratios, rates and percents. DOK 1	Apply ratio and proportionality to solve problems, including percent and simple probability DOK 2

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Level	K	1	2	3	4	5	6	7	8
Proportions	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	Write and apply ratios in mathematical and practical situations involving measurement and monetary conversions DOK 2	Write and apply proportions to solve mathematical and practical problems involving measurement and monetary conversions DOK 2	Apply ratios and proportions to calculate rates and solve mathematical and practical problems using indirect measure. DOK 2
Percentages	NA	NA	NA	NA	NA	Identify and represent ratios as comparisons of part-to-part and part-to-whole relationships, and solve problems involving ratios. DOK 1	Represent percents in various forms using numbers, pictures, models, or circle graphs and solve problems involving percentages. Greater than 100 and less than 1. DOK 2	Calculate the percentage of increase and decrease of a quantity in real-world and mathematical problems. DOK 2	Solve contextual problems using ratios, rates, or percents and verify the reasonableness of the solution. DOK 2

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Benchmark	**5.2 Operations on Numbers** Students will understand meanings of operations and how they relate to one another. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Computation Whole numbers	Use concrete objects to model simple joining and separating situations (addition and subtraction) of whole numbers related to sums of 10 or less and write corresponding number sentence. DOK 2	Use concrete objects to model the addition of two or three addends and subtraction of whole numbers related to sums less than 20 and write the corresponding number sentence DOK 2 Using concrete models or drawings and strategies based on place value, add within 100, including adding a two-digit number and a one-digit number. DOK 2	Demonstrate efficient procedures for adding and subtracting 2 and 3 digit whole numbers and explain why the procedures work on the basis of place value and number properties DOK 2	Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal "jumps" on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems DOK 2	Add and subtract whole numbers (up to five –digit number) DOK 1	Add, subtract, multiply, and divide (with and without remainders) using non-negative rational numbers. DOK 1	Model addition and subtraction of integers with physical materials and the number line. DOK 1	NA	NA

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Level	K	1	2	3	4	5	6	7	8
Meaning of Operations	NA.	Use movement on the number line to demonstrate the inverse relationship between addition and subtraction DOK 1	Model, represent, and explain multiplication (products to 81) as a rectangular array , as repeated addition and skip counting, or as equal-sized moves on the number line and division as repeated subtraction, sharing and grouping DOK 2	Apply the inverse relationship between multiplication and division (e.g., 5 x 6 = 30, 30 ÷ 6 = 5) and the relationship between multiples and factors. DOK 2	Represent multiplication of up to four-digit by one digit numbers and describe how that representation connects to the related number sentence. DOK 1	Multiply four-digit numbers by two-digit numbers (including whole numbers and decimals). DOK 1 Add, subtract, multiply, and divide fractions, decimals to hundredths, using concrete models or drawings and strategies based on place value DOK 1	Use various methods to find quotients for multi-digit division problems. And justify why the procedures work on the basis of place value and number properties DOK 2	NA	NA
Operations with fractions	NA	NA	NA	Use concrete models to add and subtract simple common fractions with the same denominator. DOK 1	Compare and order positive fractions (including positive mixed numbers) and decimals on the number line, in number sentences, and in lists DOK 1	Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators DOK 1	Apply the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and justify why they work DOK 2	NA	NA

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Level	K	1	2	3	4	5	6	7	8
Prime and composite Numbers	NA	NA	NA	Identify whole number factors and/or pairs of factors for a given whole number through 24. DOK 1	Identify factors of composite numbers less than 100 DOK 1	Identify all whole number factors and pairs of factors for a given whole number through 144 DOK 1	Identify the greatest common factor for a set of whole numbers. DOK q	Determine the Greatest Common Factor (GCF) and Least Common Multiple (LCM) of two numbers in the context of problem-solving DOK 2	Apply the concepts of Greatest Common Factor (GCF) and Least Common Multiple (LCM) to monomials with variables DOK 1
Divisibility pf Numbers	NA	NA	NA	Illustrate with manipulatives when a number is divisible by 2, 3, 5, or 10 DOK 1	Use divisibility concepts to classify numbers as prime or composite DOK 2	Model and distinguish between factor and multiple and prime and composite numbers. DOK 2	Express a whole number as a product of its prime factors, using exponents when appropriate. DOK 1	Use the concepts of number theory, including prime and composite numbers, factors, multiples, and the rules of divisibility to solve problems DOK 2	NA.

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Level	K	1	2	3	4	5	6	7	8
Odd and Even Numbers	NA	Identify odd and even numbers to 20 and determine if a set of objects has an odd or even number of elements. DOK 1	Demonstrate the relationships between odd and even numbers in addition and subtraction such as, odd + odd = even or odd – even = odd. DOK 3	Sort whole numbers into sets containing only odd numbers or only even numbers. DOK 1	NA	NA	NA	NA	NA
Mental Math	NA	Recall from memory single digit addition facts (to 9 + 9) and the corresponding subtraction facts DOK 1	Carry out addition and subtraction mentally involving: 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds DOK 2	Add or subtract with numbers less than 100 using mental arithmetic DOK 2	Recall from memory multiplication facts for numbers from 1 to 10 DOK 1	Recall from memory multiplying and dividing by 10, 100, and 1,000 DOK 1	Develop and use strategies for mental computations with non-negative whole numbers, fractions, and decimals. DOK 2	**NA**	**NA**

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Level	K	1	2	3	4	5	6	7	8
Properties of Numbers	Model meanings of operations and the relationship between addition and subtraction (e.g., identity element of addition, commutative property) using manipulatives DOK 2	Use the concept of commutative [4 + 2 = 2 + 4], associative [(4 + 3) + 7 = 4 + (3 + 7)], and identity [0 + 3 = 3] properties of addition to solve problems involving basic facts. DOK 1	Model and justify the relationship between addition and subtraction (e.g., identity element of addition, associative property, commutative property, inverse operations, fact families). DOK 2	Use and explain the operations of multiplication and division including the properties (e.g., identity element of multiplication, commutative property, property of zero, associative property, inverse operations). DOK 2	Apply models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive). DOK 1 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors DOK 1	NA	NA	NA	NA

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Benchmark	**5.3 Numerical Operations and Estimation** Students will accurately calculate and use estimation techniques, number relationships, operation rules, and algorithms; they will determine the reasonableness of answers and the accuracy of solutions.								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Estimation	Estimate the number of objects in a group of 20 or less and count to evaluate reasonableness of estimation. DOk 2	Estimate the number of objects in a group of 100 or less and count to evaluate reasonableness of estimate. DOK w	Use rounding to analyze the reasonableness of a sum or a difference. DOK 2	Apply estimation skills (rounding, benchmarks, compatible numbers) to solve and evaluate reasonableness of an answer DOK 2	Estimate solutions to problems including rounding, benchmarks, compatible numbers and evaluate the reasonableness of the solution, justify results. DOK 2	Use mental math and estimation strategies to predict the results of computations (i.e., whole numbers, addition and subtraction of fractions) and to test the reasonableness of solutions DOK 2	Use and explain estimation strategies to predict computational results with positive fractions and decimals DOK 2	Determine the reasonableness of answers involving positive fractions and decimals by comparing them to estimates DOK 2	Estimate the answer to an operation involving rational numbers based on the original numbers DOK 1
Rounding	NA	Given a number and number line/hundreds chart, identify the nearest ten DOK 1	Round numbers to the nearest 10 or 100 and identify situations in which rounding is appropriate DOK 2	Round whole numbers through 10,000 to the nearest ten, hundred, and thousand and round fractions to the nearest whole number DOK 1	Round whole numbers to 1,000,000 to any place value and round decimals to the nearest whole, 10^{th}, or 100^{th} place. DOK 1	NA	NA	NA	NA

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Level	K	1	2	3	4	5	6	7	8
Problem Solving	Create grade – appropriate story picture and story problems, solve using a variety of strategies, present and justify results DOK 2	Solve and create a story problem that matches an addition or subtraction expression or equation using physical objects, pictures, or words. DOK 2	Carry out addition and subtraction mentally involving: 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds DOK 2	Generate and solve two step addition and subtraction problems and one step multiplication problems based on practical situations DOK 2	Generate and solve addition, subtraction, multiplication, and division problems using whole numbers in practical situations DOK 2	Select, sequence, and use appropriate operations to solve multi-step word problems with whole numbers DOK 2	Solve problems involving addition, subtraction, multiplication, and division of rational numbers and express answers in simplest form DOK 1	Use the order of operations to simplify and/or evaluate whole numbers (including exponents and grouping symbols). DOK 1	Simplify and evaluate expressions using order of operations and use real number properties to justify solutions DOK 1
Problem Solving	NA	Apply strategies, including counting on, counting back, and doubling, for addition facts to at least 10. DOK 1	Apply strategies, including counting on, counting back, doubling, and halving, for addition and subtraction facts. DOK 1	Describe and show relationships between strategies and procedures for multiplying and dividing that involve addition and subtraction and explain strategies DOK 2	Select and test algorithms used in computational situations that involve multiplication and division of whole numbers and explain strategies. DOK 2	Select and/or use an appropriate operation(s) to show understanding of addition and subtraction of non-negative decimals and/or fractions DOK 2	Decide which representation (i.e., fraction or decimal) of a positive number is appropriate in a real-life situation DOK 2	Set up and solve simple percent problems using various strategies, including mental math DOK 2	Identify missing information or suggest a strategy for solving a real-life, rational-number problem DOK 2

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6.0 MEASUREMENT

Measurement provides a way to answer questions about "how many," "how much" and "how far." It is an indispensable component of business, manufacturing, art, medicine and many other aspects of daily life. We describe the sizes, capacities and values of many things, from the large distances involved in space travel, to the very small quantities in computer design and microbiology, to the varying values of currencies in international monetary exchange. All people must be able to choose an appropriate level of accuracy for a measurement; to select what measuring instruments to use and to correctly determine the measures of objects, space and time. These activities require people to be able to use standard instruments including rulers, volume and capacity measures, timers and emerging measurement technologies found in the home and workplace.

ENDURING UNDERSTANDINGS:

Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other.

ESSENTIAL QUESTIONS:

1. How are measurement and counting related?
2. How does *what* we measure affect *how* we measure?
3. How can space be defined through numbers/measurement?

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6.0 MEASUREMENT
STANDARDS AND PERFORMANCE INDICATORS

Standard	**6.0 MEASUREMENT** **Students will use concepts and tools of measurement to describe and quantify the world. Students will be able to:**								
Benchmark	**6.1 Physical Attributes** **Students will demonstrate an understanding of units of measure and measurable attributes of objects., Student will be able to:**								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Calendar	Name in order the days of the week DOK 1	Name in order the months of the year and use the calendar to identify days, weeks, months, and a year DOK 1	Recognize that there are 12 months in 1 year, 7 days in 1 week, and 24 hours in 1 day DOK 1	Recognize the number of weeks in a year, days in a year, and days in each month DOK 1	NA	NA	NA	NA	NA
Elapsed Time	Sequence events; and identify calendars and clocks as objects that measure time DOK 1	Sequence events with respect to time; e.g., yesterday, today, tomorrow, seasons DOK 1	Use elapsed time in one hour increments, beginning on the hour, to determine start, end, and elapsed time DOK 2	Use elapsed time in half-hour increments, beginning on the hour or half-hour, to determine start, end, and elapsed time DOK 2	Use elapsed time in quarter-hour increments, beginning on the quarter-hour, to determine start, end, and elapsed time DOK 2	NA	NA	NA	NA

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Level	K	1	2	3	4	5	6	7	8
Time	Tell time to the hour using digital and analog clocks DOK 1	Tell time to the hour and half-hour using digital and analog clocks DOK 1	Tell time to the nearest quarter hour and 5 minute interval using digital and analog clocks DOK 1	Tell time to the nearest minute using digital and analog clocks DOK 1	Use A.M. and P.M. appropriately in describing time DOK 1	Determine equivalent periods of time, including relationships between and among seconds, minutes, hours, days, months, and years DOK 1	NA	NA	NA
Comparison, Estimation and Conversion	Use comparative vocabulary in measurement settings (e.g., long/longer, short/shorter, more/less, hotter/colder, heavier/lighter, bigger/smaller) DOK 2	Compare, order, describe, and represent objects by length and weight DOK 2	Compare, order, and describe objects by various measurable attributes for length, weight, and temperature DOK 2	Compare, order, and describe objects by various measurable attributes for area and volume/capacity DOK 2	Estimate and convert units of measure for length, area, and weight with the same measurement system (metric) DOK 1 Estimate temperature in practical situations DOK 1	Estimate and convert units of measure for weight and volume/capacity within the same measurement system (metric) DOK 1	Estimate and compare units of measure for temperature, length, and weight/mass metric systems DOK 2	Estimate and compare corresponding units of measure for area and volume/capacity metric systems DOK 2	Estimate and convert units of measure for mass and capacity within the same measurement system (metric) DOK 1

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Benchmark	**6.2 Systems of Measurement** **Students will identify and use units, systems and processes of measurement. Students will be able to:**								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Measuring	Identify and describe measurable attributes, such as length, weight, and capacity, and use these attributes to make direct comparisons. DOK 2	Estimate and verify by measuring, length, weight, and capacity using nonstandard units (e.g., sticks, paper clips, blocks, beans) DOK 2	Select and use appropriate tools and units to measure length, time, capacity, and weight (e.g., scales for kilograms; rulers for centimeters; measuring containers for cup, and liters) thermometer in degrees Celsius DOK 1	Select and use appropriate units of measure and measure to a required degree of accuracy (to the nearest ½ unit) DOK 1	Measure length, area, weight , and temperature, to a required degree of accuracy in metric systems DOK 1	Measure volume and weight to a required degree of accuracy in the metric systems DOK 1	Explain how the size of the unit of measure used affects precision DOK 2	Estimate a measurement to the degree of precision that the tool provides DOK 2	Select an appropriate degree of precision when using measurements for calculations DOK 2

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Level	K	1	2	3	4	5	6	7	8
Units	Measure length with non-standard units; e.g., paper clips, cubes DOK 1	Identify the appropriate tool used to measure length (i.e., ruler), weight (i.e., scale), time (i.e., clock, calendar) and temperature (i.e., thermometer) DOK 1	Select the appropriate units for measuring time, length, weight, and temperature DOK 1	Select and use the appropriate standard units of measure, abbreviations, and tools for measuring length, weight, and capacity DOK 1	Use appropriate tools to determine, estimate, and compare units for measurement of weight/m)ass, area, size of angle (using the benchmark angles 45°, 90°, 180°, 270°, and 360, temperature, length, distance, and volume in metric systems and time in real-life situations. DOK 1	Select and apply appropriate units for measuring length, mass, volume, and temperature in the metric system DOK 1	Select appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area ,and volume DOK 1	Select and use appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area, and volume DOK 1	NA

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Level	K	1	2	3	4	5	6	7	8
Compare, Order, and Estimation	Determine and describe comparisons of length (longer, shorter, the same), mass (heavier, lighter, the same), and capacity (holds more, less, or about the same) using different-shaped or congruent containers, objects or figures. DOK 2	Compare and order given lengths, capacities, weights, or temperatures that are expressed in the same unit of measure DOK 2	Compare, order, and describe objects by various measurable attributes for length, weight, and temperature. DOK 2	Estimate and measure length using fractional parts to the nearest ½ unit in the Metric system. DOK 1	Estimate and measure a given object to the nearest millimeter DOK 1	Estimate and measure length to nearest ½ millimeter in the metric system DOK 1	Estimate and compare corresponding units of measure for temperature, length, and weight/mass in the metric systems DOK 2	Estimate and compare corresponding units of measure for area and volume/capacity in the metric systems. DOK 2	Estimate and convert units of measure for mass and capacity within the same measurement system (metric). DOK 1
Metric Systems	NA	NA	NA	NA	Convert capacity, weight/mass, and length within the metric system of measurement. DOK 1	Convert units within a given measurement system to include length, weight/mass, and volume. DOK 1	Convert units within a given measurement system to solve problems. DOK 1	Convert from one unit to another, perform basic operations, and solve real-world problems using standard (metric) measurements. DOK 2	NA

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Level	K	1	2	3	4	5	6	7	8
Formulas	NA	Use a variety of non-standard units to measure length DOK 1	Use non-standard units to cover a given region DOK 1	Estimate and measure perimeter and area, using links, tiles, grid paper, geoboards, and dot paper DOK 2	Describe relationships of rectangular area to numerical multiplication. DOK 2	Develop, compare, and use formulas to estimate and calculate the perimeter and area of rectangles, triangles, and parallelograms. DOK 2	Use formulas to determine the Determine the radius, diameter, and circumference of a circle and explain the relationship of circumference of a circle to its diameter, linking to pi. DOK 2	Use formulas and strategies, such as decomposition, to compute the perimeter and area of triangles, parallelograms, trapezoids, the circumference and area of circles, and find the area of more complex shapes. DOK 2	Use formulas and/or appropriate measuring tools to find length and angle measures (to appropriate levels of precision), perimeter, area, volume, and surface area of polygons, circles, spheres, cones, pyramids, and composite or irregular figures. DOK 2

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7.0 PATTERNS, FUNCTIONS, AND ALGEBRA

Algebra is the study of the patterns and symbols that make up the underlying structure of our number system. Algebraic reasoning culminates in the development of a formalized symbolic language that emerges from an early understanding of the "doing and undoing" of number operations. As young children come to understand numbers, patterns, and symbols experientially, they develop the ability to think about numbers and symbols abstractly. Very young children learn number primarily through experiences with counting. As this understanding of counting develops, children begin to use symbols to represent numbers and number sentences to represent operations. In these early years, the development of a symbolic language emerges from experiences with patterns. Students learn about number relationships such as doubles and halves, odds and evens, and multiples of twos, fives, and tens. These experiences with number patterns help students to come to the realization that numbers "behave" in predictable ways. Students represent these insights with number sentences; these number sentences form the beginning stages of symbolic reasoning.
In the intermediate grades, children look at patterns of growth and change. They make graphs and tables to look for trends and to make predictions. Algebraic reasoning is becoming more formal with students developing rules and formulas, such as the cost of a taxi ride or the area of a triangle. Students investigate how variables are related, translating number relationships into rules using variables. In order to apply these rules they solve simple equations using informal methods. Additional experiences with geometry and data reinforce this search for rules and generalizations.

By middle school, students are called upon to move fluently between a variety of models (tables, graphs, and symbolic rules) used to represent variable relationships. They are focused on the study of rates of change and the impact of slope in an equation, a graph, and a table. There is more emphasis on the proper use of symbolic language to describe variable operations. By eighth grade, students are introduced to algebraic transformations and manipulations; they are gaining experiences in demonstrating equivalence in algebraic expressions and generalizing from visual patterns and arithmetic sequences.

These students enter high school with a strong grasp of linear relationships, a formal understanding of the language of linear equations, slope and y-intercept, and a sense of how quadratic and exponential equations differ. High school students explore the increasingly formal, symbolic understandings of families of functions and symbolic transformations. They move fluently among symbolic and graphic representations with a more abstract reasoning system for expressing slope and finite differences and for demonstrating algebraic equivalence. They reason about families of functions and interpret symbolic generalizations about the behavior of standard forms. Symbolic algebra becomes the language of transformation and reasoning used in problem solving and for justification and proof in each of the four strands.
Understandings:

Enduring Understandings:
Change is fundamental to understanding functions.
Numbers or objects that repeat in predictable ways can be described or generalized.
An operation can be "undone" by its inverse.
Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities so solutions can be found.

Essential Questions:
1. How can change be described mathematically?
2. How are patterns of change related to the behavior of functions?
3. How do mathematical models/representations shape our understanding of mathematics?

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7.0 PATTERNS, FUNCTIONS, AND ALGEBRA STANDARDS and PERFORMANCE INDICATORS									
Standard	**7.0 Patterns, Functions, and Algebra** Students will use various algebraic methods to analyze, illustrate, extend, and create numerous representations (words, numbers, tables, and graphs) of patterns, functions, and algebraic relations as modeled in practical situations to solve problems, communicate, reason, and make connections within and beyond the field of mathematics.								
Benchmark	**7.1 Patterns, Relations and Functions** Students will recognize, describe and develop patterns, relations and functions. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Sorting	Sort objects into groups in one or more ways and identify which attribute was used to sort (size, shape, and color). DOK 1	Sort and classify objects by one or two attributes in more than one way DOK 1	Sort, classify, and label objects by three or more attributes in more than one way including color, size, shape, and thickness. DOK 1	NA	NA	NA	NA	NA	NA

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Level	K	1	2	3	4	5	6	7	8
Patterns	Identify, reproduce, and extend repeating patterns in visual, auditory, and physical contexts. DOK 2	Create and explain patterns using concrete objects, numbers, shapes, and colors DOK 2 Formulate, explain, and generalize patterns within and across addition and subtraction. DOK 2	Explain, analyze, and extend repeating and growing patterns DOK 2 Use number patterns to skip count by 2's, 3's, 5's, and 10's	Create, describe, and extend growing and repeating patterns with physical materials and symbols including numbers. DOK 2	Create, describe, and extend growing and repeating patterns with physical materials and symbols including numbers. DOK 1	Identify a rule for a pattern involving addition, subtraction, or multiplication DOK 1 Interpret and write a rule for a one operation function table Ex. Adding 3 DOK 2	Create and use tables and charts to extend a pattern in order to describe a rule and find missing terms in a sequence DOK 2	Use inductive reasoning to extend patterns to predict the nth term (e.g., powers and triangular numbers). DOK 2	NA
Identifying Number Patterns	Identify and describe qualitative changes (such as temperature changes – it feels hotter and quantitative changes such as temperature increases five degrees). DOK 1	Identify, describe, and explain the patterns in repeating situations (adding the same number, e.g., 2, 5, 8, 11, or skip-counting) DOK 2	Identify patterns of addition and subtraction as represented in charts and tables and in varied forms of skip-counting DOK 2	Recognize and describe patterns using objects and numbers found in tables, number charts, and charts DOK 2	Identify and describe patterns resulting from operations involving even and odd numbers (such as even + even = even) DOK 2	Identify, describe, and represent patterns and relationships in the number system, including triangular numbers and perfect squares	Identify and describe patterns represented by tables, graphs, and sequences DOK 2	Analyze and describe simple exponential number patterns (e.g., 3, 9, 27 or 3^1, 3^2, 3^3) DOK 2	Distinguish between and explain when real-life numerical patterns are linear/arithmetic (i.e., grows by addition) or exponential/geometric (i.e., grows by multiplication DOK 2

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Benchmark	**7.2 Algebraic Models** Students will represent and analyze mathematical situations and structures using algebraic symbols. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number Sentences	Use drawings and labels to record solutions of addition and subtraction problems with answers less than or equal to 10 DOK 2	Select and/or write number sentences to find the unknown in problem-solving contexts involving single-digit addition and subtraction using appropriate labels DOK 2	Select and/or write number sentences to find the unknown in problem-solving contexts involving two-digit addition and subtraction using appropriate labels DOK 2	Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit times one-digit multiplication using appropriate labels DOK 2	Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit by one-digit division using appropriate labels DOK 2	Solve problems by finding the next term or missing term in a pattern or function table using real world situations DOK 2	Determine the rule, output or input; given an input/output model using one operation, write an algebraic expression for the rule and use to identify other input/output values. DOK 2	NA	NA
Value of Variables	NA	NA	NA	NA	Determine the rule and explain how change in one variable relates to the change in the second variable, given an input/output model using two operations DOK 2	Devise a rule for an input/output function table, describing it in words and symbols. DOK 2	Represent the relationship in an input-output situation using a simple equation, graph, table, or word description DOK 2	Create tables and graphs to analyze and describe patterns DOK 2	Find the missing term in a numerical sequence or a pictorial representation of a sequence DOK 2

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Level	K	1	2	3	4	5	6	7	8
Number Sentences	NA	NA	Model, explain, and identify missing operations and missing numbers in open number sentences including number facts in addition and subtraction. DOK 2	Model, explain, and solve open number sentences including addition, subtraction, and multiplication facts. DOK 2	Model, explain, and solve open number sentences including addition, subtraction, multiplication, and division. DOK 2	Find possible solutions to an inequality involving a variable using whole numbers as a replacement set. DOK 1	Evaluate formulas and algebraic expressions using whole number values DOK 1	Evaluate formulas and algebraic expressions for given integer values DOK 1	Evaluate formulas and algebraic expressions using rational numbers DOK 1
Variables and Unknowns	NA	Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols DOK 2	Model situations and solve equations that involve the addition and subtraction of whole numbers. DOK 2	Create models for the concept of equality, recognizing that the equal sign (=) denotes equivalent terms such that 4+3=7, 4+3=6+1, or 7=5+2. DOK 2	Select the solution to an equation from a given set of numbers. DOK 1	Determine the value of variables in equations and inequalities, justifying the process. DOK 2	Solve simple equations and inequalities using guess-and-check, diagrams, properties, or inspection, explaining the process used. DOK 2	Solve equations that represent algebraic and real-world problems using multiple methods including the real number properties. DOK 2	Simplify and evaluate numerical and algebraic expressions. DOK 1
	NA	NA	NA	NA	NA	NA	Formulate algebraic expressions, equations, and inequalities to reflect a given situation. DOK 2	Formulate algebraic expressions, equations, and inequalities to reflect a given situation and vice versa. DOK 2	Model inequalities (and their solutions) on a number line. DOK 1

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Level	K	1	2	3	4	5	6	7	8
Value of Variables	NA	NA	NA	Determine the value of missing quantities or variables within equations or number sentences, and justify the process used. DOK 1	Determine the value of variables in equations; justify the process used to make the determination DOK 2.	Solve equations with whole numbers using a variety of methods, including inverse operations, mental math, and guess and check. DOK 1	Solve and graphically represent equations and inequalities in one variable with integer solutions. DOK 1	Solve and graphically represent equations and inequalities in one variable with integer solutions DOK 1	Solve and graphically represent equations and inequalities in one variable including absolute value DOK 1
Number Sentences, Expressions, and Polynomials	Identify and create, compare and describe sets of objects as more, less or equal DOK 2	Create, compare, and describe sets of objects as greater than, less than, or equal to DOK 2	Represent mathematical situations using numbers, symbols, and words and complete number sentences with the appropriate words and symbols (+, -, =) DOK 1	Complete number sentences with the appropriate words and symbols (+, -, >,< =) DOK 1	Complete number sentences with the appropriate words and symbols (+, -, >,<, ÷, x, =) DOK 1	Complete number sentences with the appropriate words and symbols including ≥, ≤, ≠ DOK 1	Write simple expressions and equations using variables to represent mathematical situations DOK 2	Simplify algebraic expressions by combining like terms DOK 1	Add and subtract binomials DOK 1

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Benchmark	7.3 Algebraic Representations Students will develop and apply mathematical models to represent and understand quantitative relationships. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Relations and Functions	NA	NA	NA	NA	NA	NA	When given a rule relating two variables create a table and represent the ordered pairs on a coordinate place DOK 2	Generate and graph a set of ordered pairs to represent a linear equation DOK 2	Identify, model, and describe linear functions DOK 2 Translate among verbal descriptions, graphic, tabular, and algebraic representations of mathematical situations DOK 2
Linear Equations and Inequalitites	NA	NA	NA	NA	NA	NA	NA	Model and solve equations using concrete and visual representations DOK 2	Solve linear equations and inequalities and represent the solution graphically DOK 2

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Benchmark	**7.4 Analysis of Change** Students will analyze change in various contexts. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Change	Describe qualitative change Ex. Changes in seasons, temperature, height, etc DOK 1	Describe quantitative changes Ex. Compare a wide variety of measurements over time (e.g., students' heights, plant growth). DOK 1	Compare and contrast the attribute changes over time in two or more qualities. DOK 2	Identify real situations and events that show change Ex. Which day had the greatest change in temperature? DOK 1	Identify, describe and generalize relationships in which quantities change proportionally DOK 2	Use a variety of methods to compare and describe situations involving constant and/or varying rates of change. DOK 2	Describe how changes in one quantity or variable result in changes in another DOK 2	Use unit rates (e.g., miles per hour, words per minutes) to solve problems DOK 2	Compare linear relationships to non-linear relationships; DOK 2

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8.0 GEOMETRY

Geometry (derived from “geo” and “metric” so, literally, the “measure of the earth”) is comprised of several important but distinct strands, all of which should be developed across grades K–12. In very young children, we build upon intuitive understandings that, in combination, make up the sense of space. These begin with simple ideas of relative position (in front of, “over there”) and culminate with two and then three dimensional coordinate geometry. Gradually, distance measurement becomes an important component of this spatial sense.

Early on, movement in both two and three dimensions becomes another component of a child’s developing spatial sense. This movement is first formalized as simple “transformations” of position accessible in the primary grades—flips, slides, and turns—and then given more formal definition and deeper exploration in the middle grades as reflections, translations, and rotations. Ultimately, transformations of position, size, and shape are embedded within the coordinate plane and described algebraically using matrices.

Visual representations play a pivotal role in supporting reasoning about geometric objects and properties. Because most representational systems, from a “flat” piece of paper to a computer screen, are two dimensional, conventions for representing three dimensional objects in two dimensions, “projections,” must be developed. A range of representational systems from paper folding (useful for emphasizing instances of congruence, for example) to Geo Logo (students must input side length and angle measure) to computer based geometric “supposers,” which support the making and testing of conjectures about geometric figures, find application across the grades.

Natural three dimensional objects can be abstracted into perfect forms, tree trunks as the prototypes of the cylinder, the arc of an ancient slingshot whirling overhead as the ideal of the circle with a given center and radius. These idealized geometrical forms are then categorized, their elements defined, and their relationship to one another determined. The classification of two and three dimensional geometric objects proceeds from an essentially perceptual and inductive basis—rectangles “look the same”—to an understanding of the set of characteristics that these objects share, for example, all rectangles have four right angles and opposite sides congruent and parallel. A primarily inductive reasoning about geometric characteristics transitions into a more deductive understanding of how one class of objects relates to another, for example, a square is a rectangle with an added constraint.

The development of a fully axiomatic system for the classification of geometric objects provides an important opportunity for high school students to develop deductive reasoning and proof.

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ENDURING INDERSTANDINGS:

1. Two- and three-dimensional objects can be described, classified, and analyzed by their attributes.
2. An object in a plane or in space can be oriented in an infinite number of ways while maintaining its size or shape.
3. An object's location on a plane or in space can be described quantitatively.
4. Linear measure, area, and volume are fundamentally different but may be related to one another in ways that permit calculation of one given the other

ESSENTIAL QUESTIONS:

1. Why do we compare contrast and classify objects?
2. How do decomposing and recomposing shapes help us build our understand of mathematics?
3. How can transformations be described mathematically?

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8.0 Geometry STANDARDS AND PERFORMANCE INDICATORS									
Standard	**8.0 GEOMETRY** The student will develop an understanding of geometric concepts and relationships as the basis for geometric modeling and reasoning to solve problems involving one-, two-, and three-dimensional figures.								
Benchmark	**8.1 Geometric Properties** Students will analyze characteristics and properties of 2 and 3 dimensional geometric shapes and develop mathematical arguments about geometric relationships. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Two-Dimensional Figures	Identify two-dimensional shapes , i.e., circle, triangle, rectangles, and squares, regardless of size or orientation DOK 1	Name, sort, and sketch two-dimensional shapes (circles, triangles, rectangles including squares) regardless of orientation. DOK 2	Describe, sketch, and compare two-dimensional shapes (rhombus, square, triangle, trapezoid, rectangle, pentagon, hexagon, octagon, and decagon) regardless of orientation. DOK 2	Describe, compare, analyze, and classify two-dimensional shapes by sides and angles DOK 2	Analyze and describe the similarities and differences between and among two dimensional geometric shapes, figures, and models using mathematical language DOK 2	Analyze and describe the characteristics of symmetry relative to classes of polygons (parallelograms, triangles, etc.). DOK 2	Identify, classify, compare, and draw regular and irregular quadrilaterals DOK 2	Identify, classify, and compare, and draw regular and irregular polygons DOK 2	Use two-dimensional representations (nets) of three-dimensional objects to describe objects from various perspectives. DOK 3

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Level	K	1	2	3	4	5	6	7	8
Three-Dimensional Figures	Identify three-dimensional figures in the environment . DOK 1	Identify, name, and describe three-dimensional objects i.e. cubes and spheres, regardless of size or orientation DOK 2	Identify, classify, and sort basic geometric figures by shape, size, and geometric attributes. E.g. cube, sphere, and cylinder, prism, pyramid, and cone. DOK 2	Identify, describe, and classify: cube, sphere, prism, pyramid, cone, and cylinder in terms of the number and shape of faces, edges, and vertices. DOK 1	Analyze the relationship between three-dimensional geometric shapes in the form of cubes, rectangular prisms, and cylinders and their two-dimensional nets. DOK 1	Predict and describe the effects of combining, dividing, and changing shapes into other shapes. DOK 2	Construct three-dimensional figures using manipulatives and generalize the relationships among vertices, faces, and edges (such as Euler's Formula). DOK 2	Construct two-dimensional representations of three-dimensional objects. DOK 2	Classify and compare three-dimensional shapes using their properties. DOK 2
Benchmark	**8.2 Transformation of** Shapes Students will apply transformations and the use of symmetry to analyze mathematical situations								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

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Congruency and Similarity	Use relative position words including before/after. far/near, and over/under to place objects DOK 1	Use position words down/up, left/right, top/bottom, and between/middle to describe the relative location of objects DOK 1	Identify congruent and similar shapes (circles, triangles, and rectangles including squares) DOK 1	Identify and create shapes that have lines of symmetry DOK 1	Identify shapes that are congruent, similar, and/or symmetrical using a variety of methods including transformational motions DOK 2	Represent concepts of congruency, similarity and/or symmetry using a variety of methods including dilation and transformational motion DOK 2	Generalize the relationship between line symmetry and rotational symmetry for two-dimensional shapes. DOK 2	Analyze geometric properties and the relationships among the properties of triangles, congruence, similarity, and transformations to make deductive arguments. DOK 3	Apply the properties of equality and proportionality To find missing attributes of congruent or similar shapes DOK 2
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Transformations	Describe the location of one object relative to another object using words such as *in*, *out*, *over*, *under*, *above*, *below*, *between*, *next to*, *behind*, and *in front of*. DOK 1	Use the directional words left, and right to describe movement. DOK 1	Use the positional and directional terms north, south, east, and west to describe location and movement. DOK 1	Describe the transformational motions of geometric figures (translation/slide, reflection/flip, and rotation/turn) DOK 1	Compare figures to determine congruence using geometric transformations (motions), such as reflections (flips), rotations (turns), and translations (slides). DOK 2	Predict the results of multiple transformations on a geometric shape when combinations of translation, reflection, and rotation are used. DOK 1	Compare, classify, and construct transformations (reflections, translations, and rotations). DOK 2	Perform transformations (rigid and non-rigid motions) on two-dimensional figures using the coordinate plane. DOK 2	Draw the results of a combination of transformations in the coordinate plane; i.e., reflections, rotations, and translations DOK 2

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Representing Figures	Represent two-dimensional geometric shapes DOK 1	Compose and decompose common two-dimensional figures. DOK 2	Predict the results of combining and subdividing polygons and circles. DOK 2	Analyze the results of combining and subdividing circles, triangles, quadrilaterals, pentagons, hexagons, and octagons. DOK 2	Represent the two-dimensional shapes trapezoids, rhombuses, and parallelograms and the three-dimensional shapes cubes, rectangular prisms, and cylinders. DOK 1	Predict and describe the effects of combining, dividing, and changing shapes into other shapes. DOK 2	Make a two-dimensional drawing of a three-dimensional figure to visualize and solve problems; e.g., those involving surface area and volume DOK 2	Make a model of a three-dimensional figure from a two-dimensional drawing. DOK 2	Use two-dimensional representations of three dimensional objects to visualize and solve problems; e.g., those involving surface area and volume DOK 2

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Level	K	1	2	3	4	5	6	7	8
Triangles	NA	NA	NA	NA	Identify, classify and draw triangles based on their properties DOK 1	Describe the characteristics of a right triangle DOK 1	Describe the characteristics of right, acute, obtuse, scalene, equilateral, and isosceles triangles and determine the area of a right triangle	Classify and construct triangles based on side and angle measurements; DOK 2	Determine the measure of the missing side of a triangle. DOK 1
	NA	NA	NA	NA	NA	Determine the degrees of the interior angles of triangles DOK 1	Find and verify the sum of the measures of interior angles of triangles DOK 1	Determine the measure of missing angles of triangles based on the Triangle Sum Theorem DOK 1	NA
	NA	NA	NA	NA	NA	NA	NA	Create an argument using the Pythagorean Theorem principles to show that a triangle is a right triangle. DOK 2	Explain the Pythagorean Theorem and apply it to solve routine and non-routine problems. DOK 3

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Level	K	1	2	3	4	5	6	7	8
Lines, Angles, and their properties	NA	NA	NA	Identify, draw, and describe horizontal, vertical, and oblique lines DOK 1	Identify, draw, label, and describe points, line segments, rays, and angles DOK 1	Identify, draw, label, and describe planes, parallel lines, intersecting lines, and perpendicular lines DOK 1	Model slope (pitch, angle of inclination) using concrete objects and practical examples DOK 2	Determine slope of a line, midpoint of a segment and the horizontal and vertical distance between two points using coordinate geometry DOK 2	Calculate slope, midpoint, and distance using equations and formulas DOK 1
Angles	NA	NA	NA	NA	Identify, draw, and classify angles, including straight, right, obtuse, and acute. DOK 1	Identify the attributes of an angle and draw angles using protractors DOK 1	Identify relationships between pairs of angles; i.e., adjacent, vertical, complementary, and supplementary DOK 1	Find missing angle measurements for parallel lines cut by a transversal(s) and for a vertex of a regular polygon. DOK 2	Locate and identify angles formed by parallel lines cut by a transversal(s) (e.g., adjacent, vertical, complementary, supplementary, corresponding, alternate interior, and alternate exterior). DOK 1

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Benchmark	**8.3 Coordinate Geometry** Students will specify locations and describe spatial relationships using coordinate geometry and other representational systems.								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Descriptions Spatial Relationships	Use the positional words near, far, below, above, beside, next to, across from, and between to describe the location of an object DOK 1	Use the positional and directional terms north, south, east, and west to describe location and movement DOK 1	Use ordered pairs to identify the locations of points in a grid; e.g., A-10 on a map DOK 1	Use coordinates to give or follow directions from one point to another on a map or grid. DOK 2	Locate and label ordered pairs in the first quadrant of the coordinate plane. DOK 1	Graph coordinates representing geometric shapes in the first quadrant DOK 1	Draw and label the components of the coordinate plane; i.e., coordinates, quadrants, origin, x- and y-axes DOK 1	Represent shapes using coordinate geometry DOK 1	Apply strategies and procedures to find the coordinates of the missing vertex of a square, rectangle, or right triangle when given the coordinates of the polygon's other vertices. DOK 2

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Benchmark	8.4 Visualization and Geometric Models Students will use visualization, spatial reasoning and geometric modeling.								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Construction of geometric figures	NA	NA	NA	NA	NA	NA	Construct circles, angles, and triangles based on given measurements using a variety of methods and tools including compass, straight edge, paper folding, and technology DOK 2	Build and sketch three-dimensional solids; e.g., using nets, manipulatives. DOK 2 Construct and identify congruent angles, parallel lines, and perpendicular lines DOK 1	Construct geometric figures using a variety of tools DOK 2
Mathematical reasoning Logic	Put events in a logical sequence DOK 1	Identify what comes next in a step by step story or event sequence DOK 1	Sort and classify objects by two or more attributes. DOK 1	Use the quantifiers all, some, and none to describe the characteristics of a set DOK 1	Use the connectors and, or, and not to describe the members of a set DOK 1	Represent relationships using Venn diagrams DOK 2	Identify counterexamples to disprove a conditional statement DOK 2	Make and test conjectures to explain observed mathematical relationships and to develop logical arguments to justify conclusions DOK 3	Represent logical relationships using conditional statements DOK 3

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9.0 DATA ANALYSIS and PROBABILITY

In the primary grades, students collect data about important questions that involve choices, “What do you like better, whales or eagles, pizza or ice cream?” They organize the results in pictographs with visuals that physically represent the choices (a picture of a whale or an ice cream cone is placed on the graph and represents the choice). Students construct their own data displays and use them to answer questions such as “which is less and which is more.” As the study of data progresses, students conduct experiments and surveys that include more categories and choices. Students are called upon to order their data using tables and tallies, and ultimately to represent both categorical and numerical data. Their use of graphs expands to include dot/line plots, and they begin to think more globally about the shape of the distribution of the data. Student experiences with probability are also based on common experiences and shared understandings of the world around them such as “how likely is it that the sun will shine tomorrow or is it likely that we will have rain tomorrow?” Their thinking about probability is categorical (likely vs. unlikely or possible vs. impossible) rather than numerical.

As they enter the intermediate grades, students begin to think more critically about one variable data sets. They grapple with the need to determine a typical, or average, value from a set of data, which leads to understanding different measures of central tendency. They pose questions and design experiments that require the collection of numerical data. In addition to pictographs, bar graphs, and line plots, they begin to use circle graphs to represent and analyze data. Student experiences with probability expand and become more formalized as they conduct experiments to determine whether a game with spinners or number cubes is “fair.” They develop ways to make organized lists and sample spaces to represent all possible outcomes, primarily focused on fair games, and use statistical tools such as tallies and relative frequencies to solve probability problems.

In middle school, students are able to take two variable data sets and compare those using measures of central tendency, stem and leaf plots, histograms and box and whisker plots. They can demonstrate that there are uses and misuses of data and how measures of central tendency may be misleading. Students are more aware of the impact of scaling on graphs and can demonstrate their understanding by building effective displays of data. They use scatter plots to investigate the relationship between two variables and use lines of best fit, where appropriate, to make predictions. Middle school experiences with probability become more sophisticated and involve the use of simulations. Students think about how to select a “fair” (random) sample. They develop an understanding of theoretical probability and how it relates to experimental probability. Well organized tree diagrams and lists are used to justify calculations of theoretical probabilities. Conceptual and procedural knowledge of probability now requires fluency with rational number and a strong grounding in proportional reasoning

In high school, while conceptual foundations for both statistics and probability continue to be developed using data drawn from experiments, students are expected to generalize and formalize their knowledge. Scatter plots and mathematical models enable predictions to be made, while mathematical reasoning addresses possible limitations for the predictions. Experimental probabilities support conjectures regarding theoretical probabilities, which can now be computed using formulas. Statistics and probability are woven together through investigations of distributions of data, including normal distributions.

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ENDURING UNDERSTANDINGS:

1. The question to be answered determines the data to be collected and how best to collect it.
2. Basic statistical techniques can be used to analyze data in the workplace.
3. The probability of an event can be used to predict the probability of future events.

ESSENTIAL QUESTIONS:

1. What is average?
2. What makes a data representation useful?
3. How does my sample affect confidence in my predication?
4. What is fair?

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9.0 DATA ANALYSIS and PROBABILITY
STANDARDS AND PERFORMANCE INDICATORS

Standard	**9.0 DATA ANALYSIS and PROBABILITY** **Students will develop an understanding of Data Analysis and Probability by solving problems in which** there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions and to present convincing arguments; and to model mathematical situations to determine the probability.								
Benchmark	**9.1 Data Representation** **Students will formulate questions that can be addressed with data and collect, organize and display relevant data to answer them. Students will be able to:**								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Organizing and Representing Data	Collect and organize data by counting and using tally marks and other symbols. DOK 1	Gather data and represent data using tallies, tables, picture graphs, and bar-type graphs. DOK 2	Use tables, pictographs, and bar graphs to represent data DOK 2	Use graphical representations including number lines, frequency tables, and pictographs to represent data DOK 1	Use a variety of graphical representations including frequency tables and plots to organize and represent data DOK 2	Use a variety of graphical representations including including line graphs, stem-and-leaf plots, histograms, and box-and-whisker plots to organize and represent data DOK 2	Use a variety of graphical representations including circle graphs and scatter plots to organize and represent data DOK 1	Use the appropriate graphical representations to organize and represent data DOK 2	Use appropriate graphical representations to organize, display, and read data. DOK 2
Formulating Questions	Identify a real life situation to gather data over time; DOK 1	Identify a real life situation to gather data over time; DOK 1	Identify real life situations to gather data over time; DOK 1	Pose questions that can be used to guide data collection, organization, and representation DOK 2	Pose questions that can be used to guide data collection, organization, and representation DOK 2	Pose questions that can be used to guide the collection of categorical and numerical data DOK 2	Formulate questions that guide the collection of data DOK 2	Formulate questions that guide the collection of data DOK 2	Formulate questions and design a study that guides the collection of data DOK 3

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Benchmark	9.2 Data Analysis **Students will select and use appropriate statistical methods to analyze data. Students will be able to:**								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Interpreting Data	Describe data by using mathematical language such as more than, less than, etc DOK 1	Analyze and interpret data by using mathematical language such as more than, less than, etc. DOK 2	Interpret data presented in circle, line, and bar graphs and answer questions about the displayed situation. DOK 2	Compare data and interpret quantities represented on tables and different types of graphs (line plots, pictographs, and bar graphs), make predictions, and solve problems based on the information. DOK 2	Compare data and interpret quantities represented on tables and graphs including line graphs, bar graphs, frequency tables, and stem-and-leaf plots to make predictions and solve DOK 2	Compare data and interpret quantities represented on tables and graphs, including line graphs, stem-and-leaf plots, histograms, and box-and-whisker plots to make predictions, and solve problems based on the information DOK 2	Interpret and explain line graphs, double bar graphs, frequency plots, stem-and-leaf plots, histograms, and box-and-whisker plots. DOK 2	Interpret graphical representations of data to describe patterns, trends, and data distributions DOK 2	Draw inferences, make conjectures and construct convincing arguments involving different effects that changes in data values have on measures of central tendency misuses of statistical or numeric information, based on data analysis of same and different sets of data DOK 3

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Level	K	1	2	3	4	5	6	7	8
Central Tendency	NA	NA	NA	NA	Determine or calculate the mode, mean/average, and range for a data set DOK 1	Collect and analyze data using mean, median and mode to determine the best statistical measure. DOK 2	Analyze and solve application problems involving measures of central tendency (mean, median, mode) and dispersion (range) from data, graphs, tables, and experiments use appropriate technology to compare two sets of data DOK 3	Determine which measure of central tendency (mean, median) provides the most useful information in a given context. DOK 2	Select and apply appropriate measures of data distribution using interquartile range and central tendency DOK 2

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Benchmark	9.3 Inferences and Predictions Students will develop and evaluate inferences and predictions that are based on data. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Inferences and Predicitons	NA	NA	NA	Record results of activities involving chance (e.g., coin flips, dice rolls) and make reasonable predictions based upon data DOK 2	Make predictions and draw conclusions from simple experiments DOK 2	Make predictions and draw conclusions based on data collected from a sample group DOK 2	Analyze various representations of a set of data to draw conclusions and make predictions. DOK 2	Interpolate and extrapolate from data to make predictions for a given set of data. DOK 2	Formulate reasonable inferences and predictions through interpolation and extrapolation of data to solve practical problems DOK 3

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Level	K	1	2	3	4	5	6	7	8
Experiments and samples	NA	NA	NA	NA	NA	NA	NA	Identify a real life situation using statistical measures (mean, median, mode, range, outliers) overtime, make a hypothesis as to the outcome; design and implement a method to collect, organize and analyze data; analyze the results to make a conclusion; evaluate the validity of the hypothesis based upon collected data, design a mode of presentation using words, graphs, models, and/or tables DOK 4	Analyze problem situations, games of chance, and consumer applications using random and non-random samplings to determine probability, make predictions, and identify sources of bias. DOK 4

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Benchmark	9.4 Probability Students will understand and apply basic concepts of probability. Students will be able to:								
Level	**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Probability Experimental Probability	Use chance devices like spinners and dice to explore concepts of probability and use tallies to record results in a table, make predictions (More likely, less likely, equally likely) based on results. DOK 2	Conduct simple experiments, record data on a tally chart or table and use the data to predict which of the events is more likely or less likely to occur if the experiment is repeated. DOK 2	Conduct simple experiments with more than two outcomes and use the data to predict which event is more, less, or equally likely to occur if the experiment is repeated. DOK 2	Develop and conduct grade-appropriate experiments using concrete objects (e.g. counters, number cubes, spinners) to determine the likeliness of events and list all outcomes. DOK 2	Design and conduct a simple probability experiment using concrete objects, examine and list all possible combinations using a tree diagram, represent the outcomes as a ratio and present the results DOK 2	Construct a sample space and make a hypothesis as to the probability of a real life situation overtime, test the prediction with experimentation, and present conclusions DOK 2	Perform simple probability events using manipulatives; predict the outcome given events using experimental and theoretical probability; express experimental and theoretical probability as a ratio, decimal or percent. DOK 2	Determine theoretical probability of an event, make and test predictions through experimentation. DOK 2	Compare the experimental and theoretical probability of a given situation (including compound probability of a dependent and independent event). DOK 2
Probability Permutations and Combinations	NA	NA	NA	NA	NA	Determine combinations and permutations of given real-world situations by multiple strategies, including creating lists. DOK 2	Determine combinations and permutations by constructing sample spaces (e.g., listing, tree diagrams, frequency distribution tables). DOK 2	Determine and explain whether a real-world situation involves permutations or combinations, then use appropriate technology to solve the problem. DOK 2	Find the number of combinations possible in mathematical and practical situations DOK 2

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Glossary for Mathematics (Source: Common Core)

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

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Associative property of multiplication. See Table 3 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team. Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See*: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

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First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form a or –a for some whole number a.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.4 Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values. Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

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Properties of operations. See Table 3 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Properties of equality. See Table 4 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

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Properties of inequality. See Table 5 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Properties of operations. See Table 3 http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space. Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or – a/b for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median M, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

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Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3,....

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[1]Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/ standards/mathglos.html, accessed March 2, 2010.

[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

[3]Adapted from Wisconsin Department of Public Instruction, op. cit.

[4]To be more precise, this defines the arithmetic mean.

[5]Adapted from Wisconsin Department of Public Instruction, op. cit.

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