AMERICAN EDUCATION REACHES OUT

AERO

COMMON CORE PLUS

# AERO
# CURRICULUM FRAMEWORK FOR MATH

**Introduction**

The *AERO Curriculum Framework for Mathematics* builds on the *Common Core State Standards for Mathematics*. The standards in this UPDATED *Framework* are the culmination of our effort to fulfill the need for pre-kindergarten through grade 12 standards aligned to the internationally benchmarked standards.

In 2009, prior to the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practice (NGA) standards development initiative, AERO revised their existing *Standards. In 2010,* The *Common Core State Standards for Mathematics* were released. AERO, then, released a crosswalk to demonstrate the similarities and differences in the documents. Recently there have been requests to up date the standards to reflect the Standards.

As specified by CCSSO and NGA, these standards are (1) research- and evidence-based, (2) aligned with college and work expectations, (3) rigorous, and (4) internationally benchmarked. The development of these standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skills, and understanding develop over time. Standards are intended to be a living work: as new and better evidence emerges, the standards will be revised accordingly.

The standards do not dictate curriculum or teaching methods. In fact, standards from different domains and clusters are sometimes closely related. What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know … should next come to learn …." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep. www.corestandards.org

Guiding Principles for Mathematics

**Guiding Principle 1: Learning**
Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

**Guiding Principle 2: Teaching**
An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

**Guiding Principle 3: Technology**
Technology is an essential tool that should be used strategically in mathematics education.

**Guiding Principle 4: Equity**
All students should have a high quality mathematics program that prepares them for college and career.

**Guiding Principle 5: Assessment**
Assessment of student learning in mathematics should take many forms to inform instruction and learning

NCTM Principles Standards for Mathematics

## AERO Mathematics Standards
## Updated 7/1/2015

| Progressions Pre K- 2 | | | | |
|---|---|---|---|---|
| **Domain: Counting** | **PreK** | **K** | **1** | **2** |
| **Know number names and the count sequence.** | AERO.PK.CC.1<br>Count verbally to 10 by ones. | AERO.K.CC.1 **DOK 1**<br>Count to 100 by ones and by tens. | AERO.1. NBT.1 **DOK 1,2**<br>Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. | AERO.2.NBT.2 **DOK 1**<br>Count within 1000; skip-count by 5s, 10s, and 100s. |
| | AERO.PK.CC.2<br>Recognize the concept of just after or just before a given number in the counting sequence up to 10. | AERO.K.CC.2 **DOK 1,2**<br>Count forward beginning from a given number within the known sequence (instead of having to begin at 1). | | |
| | AERO.PK.CC.3<br>Identify written numerals 0-10. | AERO.K.CC.3 **DOK 1**<br>Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects) | | AERO.2.NBT.3 **DOK 1,2**<br>Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. |
| **Count to tell the number of objects.** | AERO.PK.CC.4<br>Understand the relationship between numbers and quantities; connect counting to cardinality. | AERO.K.CC.4 **DOK 2**<br>Understand the relationship between numbers and quantities; connect counting to cardinality | | |
| | AERO.PK.CC.4a.<br>When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object | AERO.K.CC.4a **DOK 2**<br>When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. | | |

| **Domain: Counting** | **PreK** | **K** | **1** | **2** |
|---|---|---|---|---|
| **Count to tell the number of objects.** | AERO.PK.CC.4b<br>Recognize that the last number name said tells the number of objects counted. | AERO.K.CC.4b **DOK 2**<br>Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. | | |
| | AERO.PK.CC.4c<br>Recognize that each successive number name refers to a quantity that is one larger. | AERO.K.CC.4c **DOK 2**<br>Understand that each successive number name refers to a quantity that is one larger. | | |
| | AERO.PK.CC.5<br>Represent a number (0-5, then to 10) by producing a set of objects with concrete materials, pictures, and/or numerals (with 0 representing a count of no objects). | AERO.K.CC.5 **DOK 2**<br>Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects. | | |
| | AERO.PK.CC.6<br>Recognize the number of objects in a set without counting (Subitizing). (Use 0-5 objects) | | | |

| Domain: Counting | PreK | K | 1 | 2 |
|---|---|---|---|---|
| **Compare numbers.** | AERO.PK.CC.7<br>Explore relationships by comparing groups of objects up to 10, to determine greater than/more or less than, and equal to/same Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies (includes groups with up to 5 objects). | AERO.K.CC.6 **DOK 2**<br>Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies<br><br>AERO.K.CC.7 **DOK 1.2**<br>Compare two numbers between 1 and 10 presented as written numerals. | AERO.1.NBT.3 **DOK 2**<br>Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. | AERO.2.NBT.4 **DOK 2**<br>Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. |

| Domain: Numbers in Base Ten | PreK | K | 1 | 2 |
|---|---|---|---|---|
| **Work with numbers 11-19 to gain foundations for place value** | AERO.PK.NBT.1<br>Investigate the relationship between ten ones and ten | AERO.K.NBT.1 **DOK 2**<br>Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. | AERO.1.NBT.2 **DOK 2**<br>Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: | AERO.2.NBT.1 **DOK 2**<br>Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: |
| | | | AERO.1.NBT.2a **DOK 2**<br>10 can be thought of as a bundle of ten ones — called a "ten." | AERO.2.NBT.1a **DOK 2**<br>100 can be thought of as a bundle of ten tens — called a "hundred." |
| | | | AERO.1.NBT.2b **DOK 2**<br>The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. | |
| | | | AERO.1.NBT.2c **DOK 2**<br>The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones) | AERO.2.NBT.1b **DOK 2**<br>The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). |

| **Domain: Numbers in Base Ten** | **PreK** | **K** | **1** | **2** |
|---|---|---|---|---|
| **Use place value understanding and properties of operations to add and subtract.** | | | AERO.1.NBT.4 **DOK 1,2,3** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. | AERO.**2.NBT.5** **DOK 1,2** **Fluently** add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. |
| | | | AERO.1.NBT.5 **DOK 2,3** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. | AERO.2.NBT.8 **DOK 2** Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. |

| **Domain: Numbers in Base Ten** | **PreK** | **K** | **1** | **2** |
|---|---|---|---|---|
| **Use place value understanding and properties of operations to add and subtract.** | | | AERO.1.NBT.6 **DOK 2,3** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. | |

| **Domain: Operations Algebraic Thinking** | **PreK** | **K** | **1** | **2** |
| --- | --- | --- | --- | --- |
| **Represent and solve problems involving addition and subtraction.** | | | | AERO.2.NBT.7 **DOK 2**<br>Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. |
| **Represent and solve problems involving addition and subtraction.** | | | | AERO.2.NBT.6 **DOK 2**<br>Add up to four two-digit numbers using strategies based on place value and properties of operations. |
| | | | | AERO.2.NBT.9 **DOK 3**<br>Explain why addition and subtraction strategies work, using place value and the properties of operations. |

| **Domain: Operations Algebraic Thinking** | **PreK** | **K** | **1** | **2** |
|---|---|---|---|---|
| **Understand addition, and understand subtraction.** | AERO.PK.OA.1<br>Explore addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, or verbal explanations. | AERO.K.OA.1 **DOK 2**<br>Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. | AERO.1.OA.1 **DOK 2**<br>Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem | AERO.2.OA.1 **DOK 2**<br>Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |
| | | AERO.K.OA.2 **DOK 2**<br>Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. | AERO.1.OA.2 **DOK 2**<br>Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. | |
| | AERO.PK.OA.2<br>Decompose quantity (less than or equal to 5, then to 10) into pairs in more than one way (e.g., by using objects or drawings). | AERO.K.OA.3 **DOK 2.3**<br>Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1) | | |

| **Domain: Operations Algebraic Thinking** | **PreK** | **K** | **1** | **2** |
|---|---|---|---|---|
| **Understand addition, and understand subtraction.** | AERO.PK.OA.3<br>For any given quantity from (0 to 5, then to 10) find the quantity that must be added to make 5, then to 10, e.g., by using objects or drawings. | AERO.K.OA.4 **DOK 2**<br>For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. | | |
| | | AERO.K.OA.5 **DOK 1**<br>**Fluently add and subtract within 5.** | AERO.1.OA.6 **DOK 1,2**<br>Add and subtract within 20, **demonstrating fluency for addition and subtraction within 10**. Use strategies such as counting on; making ten *(e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14*); decomposing a number leading to a ten (*e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9*); using the relationship between addition and subtraction (*e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4*); and creating equivalent but easier or known sums (*e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13*). | AERO.2.OA.2 **DOK 1**<br>**Fluently add and subtract within 20** using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. |

| **Domain: Operations Algebraic Thinking** | **PreK** | **K** | **1** | **2** |
|---|---|---|---|---|
| Understand and apply properties of operations and the relationship between addition and subtraction | | | AERO.1.OA.3 **DOK 2**<br>Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.) | |
| | | | AERO.1.OA.4 **DOK 2**<br>Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8 | |
| Add and subtract within 20. | | | AERO.1.OA.5 **DOK 1,2**<br>Relate counting to addition and subtraction (e.g., by counting on 2 to add 2) | |

| Domain: Operations Algebraic Thinking | PreK | K | 1 | 2 |
|---|---|---|---|---|
| Work with addition and subtraction equations. | | | AERO.1.OA.7 **DOK 3**<br>Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2. | |
| | | | AERO.1.OA.8 **DOK 2**<br>Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _<br>. | |

| Domain: Operations Algebraic Thinking | PreK | K | 1 | 2 |
|---|---|---|---|---|
| Work with equal groups of objects to gain foundations for multiplication. | | | | AERO.2.OA.3 **DOK 2**<br>Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. |
| | | | | AERO.2.OA.4 **DOK 2**<br>Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. |

| Domain: Measurement and Data | PreK | K | 1 | 2 |
|---|---|---|---|---|
| Describe and compare measurable attributes | AERO.PK.MD.1<br>Describe measurable attributes of objects, such as length or weight. | AERO.K.MD.1 **DOK 2**<br>Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object | . | |
| Measure lengths indirectly and by iterating length units | | | AERO.1.MD.1 **DOK 2,3**<br>Order three objects by length; compare the lengths of two objects indirectly by using a third object | AERO.2.MD.1 **DOK 1**<br>Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. |
| | | | AERO.1.MD.2 **DOK 1,2**<br>Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps | AERO.2.MD.2 **DOK 2,3**<br>Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. |
| | | | | AERO.2.MD.3 **DOK 2**<br>Estimate lengths using units of inches, feet, centimeters, and meters. |
| | | | | AERO.2.MD.4 **DOK 1, 2**<br>Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. |

| Domain: Measurement and Data | PreK | K | 1 | 2 |
| --- | --- | --- | --- | --- |
| Relate addition and subtraction to length | | | | AERO.2.MD.5 **DOK 2**<br>Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. |
| | | | | AERO.2.MD.6 **DOK 1,2**<br>Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. |
| | AERO.PK.MD.2<br>Directly compare two objects with a measurable attribute in common, using words such as longer/shorter; heavier/lighter; or taller/shorter. | AERO.K.MD.2 **DOK 2**<br>Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter. | | |

| Domain: Measurement and Data | PreK | K | 1 | 2 |
|---|---|---|---|---|
| Tell and write time. | | | AERO.1.MD.3 **DOK 1**<br>Tell and write time in hours and half-hours using analog and digital clocks. | AERO.2.MD.7 **DOK 1**<br>Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. |
| | | | | AERO.2.MD.8 **DOK 2**<br>Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? |
| Classify objects and count the number of objects in each category. | AERO.PK.MD.3<br>Sort objects into given categories | AERO.K.MD.3 **DOK 1,2**<br>Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. | | |
| Represent and interpret data. | AERO.PK.MD.4<br>Compare categories using words such as greater than/more, less than, and equal to/same. | | AERO.1.MD.4 **DOK 2,3**<br>Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. | AERO.2.MD.9 **DOK 2**<br>Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. |

| Domain: Geometry | PreK | K | 1 | 2 |
| --- | --- | --- | --- | --- |
| Represent and interpret data. | | | | AERO.2.MD.10 **DOK 2**<br>Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems1 using information presented in a bar graph. |
| Identify and describe shapes | AERO.PK.G.1<br>Match like (congruent and similar) shapes. | AERO.K.G.1 **DOK 1,2**<br>Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to | | |
| | AERO.PK.G.2<br>Group the shapes by attributes. | AERO.K.G.2 **DOK 1**<br>Correctly name shapes regardless of their orientations or overall size. | | |
| | AERO.PK.G.3<br>Correctly name shapes (regardless of their orientations or overall size). | AERO.K.G.3 **DOK 1**<br>Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). | | |

| Domain: Geometry | PreK | K | 1 | 2 |
|---|---|---|---|---|
| **Analyze, compare, create, and compose shapes.** | AERO.PK.G.4<br>Describe three-dimensional objects using attributes. | AERO.K.G.4 **DOK 2,3**<br>Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). | AERO.1.G.1 **DOK 2**<br>Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes. | AERO.2.G.1 **DOK 1, 2**<br>Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.1 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. |
| | AERO.PK.G.5<br>Describe three-dimensional objects using attributes. | AERO.K.G.5 **DOK 2,3**<br>Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. | AERO.1.G.2 **DOK 2,3**<br>Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape | |
| | AERO.PK.G.6<br>Compose and describe structures using three-dimensional shapes. Descriptions may include shape attributes, relative position, etc | AERO.K.G.6 **DOK 2,3**<br>Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle? | | |

| Domain: Geometry | PreK | K | 1 | 2 |
|---|---|---|---|---|
| Analyze, compare, create, and compose shapes. | | | AERO.1.G.3 **DOK 1,2**<br>Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares. | AERO.2.G.2 **DOK 2**<br>Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.<br>AERO.2.G.3 **DOK 2,3**<br>Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape. |

| Mathematical Practices | PreK/K | 1 | 2 |
|---|---|---|---|
| **1. Make sense of problems and persevere in solving them.** | Use both verbal and nonverbal means, these students begin to explain to themselves and others the meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. | Explain to themselves the meaning of a problem and look for ways to solve it.<br><br>May use concrete objects or pictures to help them conceptualize and solve problems.<br><br>Are willing to try other approaches. | In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them.<br><br>Students explain to themselves the meaning of a problem and look for ways to solve it.<br><br>They may use concrete objects or pictures to help them conceptualize and solve problems.<br><br>They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach. |
| **2. Reason abstractly and quantitatively.** | Begin to use numerals to represent specific amount (quantity)<br><br>Begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation<br><br>Begin to understand how symbols (+, -, =) are used to represent quantitative ideas in a written format. | Recognize that a number represents a specific quantity.<br><br>Connect the quantity to written symbols.<br><br>Create a representation of a problem while attending to the meanings of the quantities. | Younger students recognize that a number represents a specific quantity.<br><br>They connect the quantity to written symbols.<br><br>Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities.<br><br>Second graders begin to know and use different properties of operations and also relate addition and subtraction to length. |

| **Mathematical Practices** | **PreK/K** | **1** | **2** |
| --- | --- | --- | --- |
| **3. Construct viable arguments and critique the reasoning of others.** | Begin to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations.<br><br>Begin to learn how to express opinions, become skillful at listening to others, describe their reasoning and respond to others' thinking and reasoning.<br><br>Begin to develop the ability to reason and analyze situations as they consider questions such as, "Are you sure…?" , "Do you think that would happen all the time…?", and "I wonder why…?" | Construct arguments using concrete referents, such as objects, pictures, drawings, and actions.<br><br>Explain their own thinking and listen to others' explanations.<br><br>Decide if the explanations make sense and ask questions. | Construct arguments using concrete referents, such as objects, pictures, drawings, and actions.<br><br>Explain their own thinking and listen to others' explanations.<br><br>Decide if the explanations make sense and ask appropriate questions. |
| **4. Model with mathematics.** | Begin to experiment with representing real-life problem situations in multiple ways such as with numbers, words (mathematical language), drawings, objects, acting out, charts, lists, and number sentences. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc.<br><br>Connect the different representations and explain the connections. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc.<br><br>Connect the different representations and explain the connections.<br><br>Able to use all representations as needed. |

| Mathematical Practices | PreK/K | 1 | 2 |
|---|---|---|---|
| **5. Use appropriate tools strategically.** | Begin to explore various tools and use them to investigate mathematical concepts. Through multiple opportunities to examine materials<br><br>Experiment and use both concrete materials (e.g. 3- dimensional solids, connecting cubes, ten frames, number balances) and technological materials (e.g., virtual manipulatives, calculators, interactive websites) to explore mathematical concepts. | Decide when certain tools might be helpful when solving a mathematical problem. *For example , first graders decide it might be best to use colored chips to model an addition problem.* | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. *For example, second graders may decide to solve a problem by drawing a picture rather than writing an equation.* |
| **6. Attend to precision.** | Begin to express their ideas and reasoning using words.<br><br>Begin to describe their actions and strategies more clearly, understand and use grade-level appropriate vocabulary accurately, and begin to give precise explanations and reasoning regarding their process of finding solutions. | Use clear and precise language in their discussions with others and when they explain their own reasoning. | Use clear and precise language in their discussions with others<br><br>Explain their own reasoning.<br>. |

| **Mathematical Practices** | **PreK/K** | **1** | **2** |
|---|---|---|---|
| **7. Look for and make use of structure. (Deductive Reasoning)** | Begin to look for patterns and structures in the number system and other areas of mathematics. | Begin to discern a pattern or structure. *For example, if students recognize 12 + 3 = 15, then they also know 3 + 12 = 15. (Commutative property of addition.) To add 4 + 6 + 4, the first two numbers can be added to make a ten, so 4 + 6 + 4 = 10 + 4 = 14.* | Look for patterns. *For example , they adopt mental math strategies based on patterns (making ten, fact families, doubles).* |
| **8. Look for and express regularity in repeated reasoning. (Inductive Reasoning)** | Begin to notice repetitive actions in geometry, counting, comparing, etc. | Notice repetitive actions in counting and computation, etc.<br><br>Continually check their work by asking themselves, "Does this make sense?" | Notice repetitive actions in counting and computation, etc.<br><br>Look for shortcuts, when adding and subtracting, such as rounding up and then adjusting the answer to compensate for the rounding.<br><br>Continually check their work by asking themselves, "Does this make sense?" |

# AERO Mathematics Standards
# Updated 7/1/2015

| Progressions 3-5 | | | |
|---|---|---|---|
| **Domain: Number and Operations in Base Ten** | 3 | 4 | 5 |
| **Use place value understanding and properties of operations to perform multi-digit arithmetic** | AERO. 3.NBT.1 **DOK 1**<br>Use place value understanding to round whole numbers to the nearest 10 or 100. | AERO. 4.NBT.3 **DOK 1**<br>Use place value understanding to round multi-digit whole numbers to any place | AERO. 5.NBT.4 **DOK 1**<br>Use place value understanding to round decimals to any place. |
| | AERO. 3.NBT.2 **DOK 1,2**<br>**Fluently** add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. | AERO. 4.NBT.4 **DOK 1**<br>**Fluently** add and subtract multi-digit whole numbers using the standard algorithm. | AERO. 5.NBT.5 **DOK 1**<br>**Fluently** multiply multi-digit whole numbers using the standard algorithm. |
| | AERO. 3.NBT.3 **DOK 1,2**<br>Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (*e.g., 9 × 80, 5 × 60*) using strategies based on place value and properties of operations. | AERO. 4.NBT.5 **DOK 1,2**<br>Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. | AERO. 5.NBT.2 **DOK 1,2**<br>Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. |
| | AERO. 3.OA.2 **DOK 1,2**<br>Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* | AERO. 4.NBT.6 **DOK 1,2**<br>Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. | AERO. 5.NBT.6 **DOK 1,2**<br>Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |

| Domain: Number and Operations in Base Ten | 3 | 4 | 5 |
|---|---|---|---|
| **Use place value understanding and properties of operations to perform multi-digit arithmetic** | . | AERO. 4.NF.5 **DOK 1**<br>Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* | AERO. 5.NBT.7 **DOK 1,2,3**<br>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. |
| | | AERO. 4.NF.6 **DOK 1**<br>Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* | |
| | | AERO. 4.NBT.1 **DOK 1**<br>Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* | AERO. 5.NBT.1 **DOK 1**<br>Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. |
| | | | AERO. 5.NBT.3 **DOK 1**<br>Read, write, and compare decimals to thousandths. |
| | | | AERO. 5.NBT.3a **DOK 1**<br>Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, *e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).* |
| | | | AERO. 5.NBT.3a **DOK 1**<br>Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, *e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).* |

| Domain: Number and Operations in Base Ten | 3 | 4 | 5 |
|---|---|---|---|
| **Generalize place value understanding for multi-digit whole numbers and decimals to hundredths** | | AERO. 4.NF.7 **DOK 1,2,3**<br>Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. | AERO. 5.NBT.3b **DOK 1**<br>Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons |
| **Represent and solve problems involving multiplication and division.** | AERO. 3.OA.4 **DOK 1,2**<br>Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?* | | AERO. 5.OA.2 **DOK 1,2**<br>Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* |
| | AERO. 3.OA.6 **DOK 1,2**<br>Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* | | |
| | AERO. 3.OA.3 **DOK 1,2**<br>Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, *e.g., by using drawings and equations with a symbol for the unknown number to represent the problem* | | |

| **Domain: Operations and Algebraic Thinking** | 3 | 4 | 5 |
|---|---|---|---|
| **Understand properties of multiplication and the relationship between multiplication and division.** | AERO. 3.OA.5 **DOK 1,2**<br>Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* | | AERO. 5.OA.1 **DOK 1**<br>Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols |
| | AERO. 3.OA.1 **DOK 1,2**<br>Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7* | AERO. 4.OA.1 **DOK 1,2**<br>Interpret a multiplication equation as a comparison, *e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations* | |
| **Multiply and divide within 100.** | AERO. 3.OA.7 **DOK 1,2**<br>**Fluently** multiply and divide within 100, using strategies such as the relationship between multiplication and division *(e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8*) or properties of operations. **By the end of Grade 3, know from memory all products of two one-digit numbers.** | | |

| **Domain: Operations and Algebraic Thinking** | 3 | 4 | 5 |
|---|---|---|---|
| **Solve problems involving the four operations, and identify and explain patterns in arithmetic** | AERO. 3.OA.8 **DOK 1,2,3**<br>Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding | AERO. 4.OA.3 **DOK 1,2,3**<br>Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. | |
| | AERO. 3.OA.9 **DOK 1,2,3**<br>Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* | AERO. 4.OA.2 **DOK 1,2**<br>Multiply or divide to solve word problems involving multiplicative comparison, e.*g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison*. | |

p

| Domain: Operations and Algebraic Thinking | 3 | 4 | 5 |
|---|---|---|---|
| **Gain familiarity with factors and multiples.** | | AERO. 4.OA.4 **DOK 1**<br>Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. | |
| **Generate and analyze patterns.** | | AERO. 4.OA.5 **DOK 1,2**<br>Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. | AERO. 5.OA.3 **DOK 1,2**<br>Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* |

| **Domain: Numbers and Operations-Fractions** | 3 | 4 | 5 |
|---|---|---|---|
| **Develop understanding of fractions as numbers.** | AERO. 3.NF.1 **DOK 1,2**<br>Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. | AERO. 4.NF.1 **DOK 1,2,3**<br>Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions | |
| | AERO. 3.NF.2 **DOK 1,2**<br>Understand a fraction as a number on the number line; represent fractions on a number line diagram. | | |
| | AERO. 3.NF.2a **DOK 1,2**<br>Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. | | |
| | AERO. 3.NF.2b **DOK 1,2**<br>Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line | | |
| | AERO. 3.NF.3 **DOK 1,2,3**<br>Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. | | |

| Domain: Numbers and Operations-Fractions | 3 | 4 | 5 |
|---|---|---|---|
| **Develop understanding of fractions as numbers.** | AERO. 3.NF.3a **DOK 1,2,3**<br>Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line | | |
| | AERO. 3.NF.3b **DOK 1,2.3**<br>Recognize and generate simple equivalent fractions, *e.g., 1/2 = 2/4, 4/6 = 2/3.* Explain why the fractions are equivalent, *e.g., by using a visual fraction model.* | | |
| | AERO. 3.NF.3c **DOK 1,2,3**<br>Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: *Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* | | |
| | AERO. 3.NF.3d **DOK 1,2.3**<br>Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. | AERO. 4.NF.2 **DOK 1,2,3**<br>Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. | |

| **Domain: Numbers and Operations-Fractions** | 3 | 4 | 5 |
|---|---|---|---|
| **Build fractions from unit fractions.** | | AERO. 4.NF.3 **DOK 1,2,3**<br>Understand a fraction a/b with a > 1 as a sum of fractions 1/b. | |
| **Use equivalent fractions as a strategy to add and subtract fractions.** | | AERO. 4.NF.3a **DOK 1,2,3**<br>Understand addition and subtraction of fractions as joining and separating parts referring to the same whole | AERO. 5.NF.1 **DOK 1**<br>Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* |
| | | AERO. 4.NF.3b **DOK 1,2,3**<br>Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions*, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* | |
| | | AERO. 4.NF.3c **DOK 1,2,3**<br>Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. | |

| Domain: Numbers and Operations-Fractions | 3 | 4 | 5 |
|---|---|---|---|
| **Use equivalent fractions as a strategy to add and subtract fractions.** | | AERO. 4.NF.3d **DOK 1,2,3**<br>Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. | AERO. 5.NF.2 **DOK 1,2,3**<br>Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. |
| **Apply and extend previous understandings of multiplication and division.** | | AERO. 4.NF.4 **DOK 1,2**<br>Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. | AERO. 5.NF.4 **DOK 1,2**<br>Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. |
| | | AERO. 4.NF.4a **DOK 1,2**<br>Understand a fraction a/b as a multiple of 1/b. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* | |
| | | AERO. 4.NF.4b **DOK 1,2**<br>Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* | AERO. 5.NF.4a **DOK 1,2**<br>Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) |

| **Domain: Numbers and Operations-Fractions** | 3 | 4 | 5 |
|---|---|---|---|
| **Apply and extend previous understandings of multiplication and division.** | | AERO. 4.NF.4c **DOK 1,2**<br>Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* | AERO. 5.NF.3 **DOK 1,2**<br>Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, *e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* |
| | | | AERO. 5.F.4b **DOK 1,2**<br>Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas |
| | | | AERO. 5.F.5a **DOK 1,2,3**<br>Interpret multiplication as scaling (resizing), by Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. |

| Domain: Numbers and Operations-Fractions | 3 | 4 | 5 |
|---|---|---|---|
| **Apply and extend previous understandings of multiplication and division.** | | | AERO. 5.NF.5b **DOK 1,2,3**<br>Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1. |
| | | | AERO. 5.NF.6 **DOK 1,2**<br>Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. |
| | | | AERO. 5.NF.7 **DOK 1,2**<br>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions |
| | | | AERO. 5.NF.7a **DOK 1,2**<br>Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. |

| **Domain: Numbers and Operations-Fractions** | 3 | 4 | 5 |
|---|---|---|---|
| **Apply and extend previous understandings of multiplication and division** | | | AERO. 5.NF.7b **DOK 1,2**<br>Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* |
| | | | AERO. 5.NF.7c **DOK 1,2**<br>Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?* |

| **Domain: Measurement and Data** | 3 | 4 | 5 |
| --- | --- | --- | --- |
| **Solve problems involving measurement and estimation.** | AERO. 3.MD.1 **DOK 1,2**<br>Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, *e.g., by representing the problem on a number line diagram* | | |
| | AERO. 3.MD.2 **DOK 1,2**<br>Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, *e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.* | | |
| **Solve problems involving measurement and conversion of measurements**. | | AERO. 4.MD.1 **DOK 1**<br>Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...* | AERO. 5.MD.1 **DOK 1,2**<br>Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. |

| Domain: Measurement and Data | 3 | 4 | 5 |
|---|---|---|---|
| **Solve problems involving measurement and conversion of measurements** | | AERO. 4.MD.2 **DOK 1,2**<br>Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. | |
| | | AERO. 4.MD.3 **DOK 1,2**<br>Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* | |
| **Represent and interpret data.** | AERO. 3.MD.3 **DOK 1,2**<br>Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* | AERO. 4.MD.4 **DOK 1,2**<br>Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.* | AERO. 5.NF.2 **DOK 1,2,3**<br>Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. |

| Domain: Measurement and Data | 3 | 4 | 5 |
| --- | --- | --- | --- |
| **Represent and interpret data** | AERO. 3.MD.4 **DOK 2**<br>Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. | | |
| **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**<br><br>**Geometric measurement: understand concepts of volume.** | AERO. 3.MD.5 **DOK 1,2**<br>Recognize area as an attribute of plane figures and understand concepts of area measurement. | | AERO. 5.MD.3 **DOK 1**<br>Recognize volume as an attribute of solid figures and understand concepts of volume measurement. |
| | AERO. 3.MD.5a **DOK 1,2**<br>A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. | | AERO. 5.MD.3a **DOK 1**<br>A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. |
| | AERO. 3.MD.5b **DOK 1,2**<br>A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. | | AERO. 5.MD.3b **DOK 1**<br>A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. |
| | AERO.3.MD.6 **DOK 1,2**<br>Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units) | | AERO.5.MD.4 **DOK 1,2**<br>Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. |

| **Domain: Measurement and Data** | **3** | **4** | **5** |
|---|---|---|---|
| **Geometric measurement: understand concepts of volume.** | AERO. 3.MD.7 **DOK 1,2**<br>Relate area to the operations of multiplication and addition. | | AERO. 5.MD.5 **DOK 1,2**<br>Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume |
| | AERO. 3.MD.7a **DOK 1,2**<br>Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. | | AERO. 5.MD.5a **DOK 1,2**<br>Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. |
| | AERO. 3.MD.7b **DOK 1,2**<br>Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning | | AERO. 5.MD.5b **DOK 1,2**<br>Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. |
| | AERO. 3.MD.7c **DOK 1,2**<br>Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. | | |

| **Domain: Measurement and Data** | **3** | **4** | **5** |
|---|---|---|---|
| **Geometric measurement: understand concepts of volume.** | AERO. 3.MD.7d **DOK 1,2**<br>Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. | | AERO. 5.MD.5c **DOK 1,2**<br>Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.<br><br>AERO. 5.MD.3 **DOK 1**<br>Recognize volume as an attribute of solid figures and understand concepts of volume measurement. |
| **Geometric measurement: recognize perimeter.** | AERO. 3.MD.8 **DOK 1,2**<br>Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. | | |
| **Geometric measurement: understand concepts of angle and measure angles.** | | AERO. 4.MD.5 **DOK 1**<br>Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: | |

| **Domain: Measurement and Data** | **3** | **4** | **5** |
|---|---|---|---|
| **Geometric measurement: understand concepts of angle and measure angles.** | | AERO. 4.MD.5a **DOK 1**<br>An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. | |
| | | AERO. 4.MD.5b **DOK 1**<br>An angle that turns through n one-degree angles is said to have an angle measure of n degrees. | |
| | | AERO. 4.MD.6 **DOK 1**<br>Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. | |
| | | AERO. 4.MD.7 **DOK 1,2**<br>Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, *e.g., by using an equation with a symbol for the unknown angle measure.* | |

| Domain: Geometry | 3 | 4 | 5 |
|---|---|---|---|
| Reason with shapes and their attributes. | AERO. 3.G.1 **DOK 1,2**<br>Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. | | AERO. 5.G.3 **DOK 1,2**<br>Classify two-dimensional figures into categories based on their properties. |
| | AERO. 3.G.2 **DOK 1,2**<br>Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.* | | |
| | | AERO. 4.G.1 **DOK 1**<br>Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. | |
| | | AERO. 4.G.2 **DOK 1,2**<br>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. | |
| | | AERO. 4.G.2 **DOK 1,2**<br>Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. | |

| Domain: Geometry | 3 | 4 | 5 |
|---|---|---|---|
| **Graph points on the coordinate plane to solve real-world and mathematical problems.** | | AERO. 4.G.3 **DOK 1**<br>Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. | |
| | | | .AERO. 5.G.1 **DOK 1**<br>Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate). |
| | | | AERO. 5.G.2 **DOK 1,2**<br>Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. |
| | | | AERO. 5.G.4 **DOK 1,2**<br>Classify two-dimensional figures in a hierarchy based on properties |

| Mathematical Practices | 3 | 4 | 5 |
| --- | --- | --- | --- |
| **1. Make sense of problems and persevere in solving them.** | Explain to themselves the meaning of a problem and look for ways to solve it.<br><br>May use concrete objects or pictures to help them conceptualize and solve problems.<br><br>May check their thinking by asking themselves, "Does this make sense?"<br><br>Listen to the strategies of others and will try different approaches.<br><br>Will use another method to check their answers. | Know that doing mathematics involves solving problems and discussing how they solved them.<br><br>Explain to themselves the meaning of a problem and look for ways to solve it.<br><br>May use concrete objects or pictures to help them conceptualize and solve problems.<br><br>May check their thinking by asking themselves, "Does this make sense?"<br><br>Listen to the strategies of others and will try different approaches.<br><br>Will use another method to check their answers. | Solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers.<br><br>Solve problems related to volume and measurement conversions.<br><br>Seek the meaning of a problem and look for efficient ways to represent and solve it.<br><br>Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". |

| Mathematical Practices | 3 | 4 | 5 |
| --- | --- | --- | --- |
| **2. Reason abstractly and quantitatively.** | Recognize that a number represents a specific quantity.<br><br>Connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. | Recognize that a number represents a specific quantity.<br><br>Connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.<br><br>Extend this understanding from whole numbers to their work with fractions and decimals.<br><br>Write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. | Recognize that a number represents a specific quantity.<br><br>Connect quantities to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.<br><br>Extend this understanding from whole numbers to their work with fractions and decimals.<br><br>Write simple expressions that record calculations with numbers and represent or round numbers using place value concepts. |

| Mathematical Practices | 3 | 4 | 5 |
|---|---|---|---|
| **3. Construct viable arguments and critique the reasoning of others.** | May construct arguments using concrete referents, such as objects, pictures, and drawings.<br><br>Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"<br><br>Explain their thinking to others and respond to others' thinking. | May construct arguments using concrete referents, such as objects, pictures, and drawings.<br><br>Explain their thinking and make connections between models and equations.<br><br>Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"<br><br>Explain their thinking to others and respond to others' thinking. | Construct arguments using concrete referents, such as objects, pictures, and drawings.<br><br>Explain calculations based upon models and properties of operations and rules that generate patterns.<br><br>Demonstrate and explain the relationship between volume and multiplication.<br><br>Refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?"<br><br>Explain their thinking to others and respond to others' thinking. |

| Mathematical Practices | 3 | 4 | 5 |
|---|---|---|---|
| **4. Model with mathematics.** | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc.<br><br>•Connect the different representations and explain the connections.<br><br>Evaluate their results in the context of the situation and reflect on whether the results make sense. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc.<br><br>Connect the different representations and explain the connections.<br><br>Use all of these representations as needed.<br><br>Evaluate their results in the context of the situation and reflect on whether the results make sense. | Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc.<br><br>Connect the different representations and explain the connections.<br><br>Use all of these representations as needed.<br><br>Evaluate their results in the context of the situation and whether the results make sense.<br><br>Evaluate the utility of models to determine which models are most useful and efficient to solve problems. |

| Mathematical Practices | 3 | 4 | 5 |
|---|---|---|---|
| **5. Use appropriate tools strategically.** | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. *For EXAMPLE, they may use graph paper to find all the possible rectangles that have a given perimeter.*<br><br>Compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. *For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles.*<br><br>Use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. | Consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. *For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions.*<br><br>Use graph paper to accurately create graphs and solve problems or make predictions from real world data. |
| **6. Attend to precision.** | Use clear and precise language in their discussions with others and in their own reasoning.<br><br>Are careful about specifying units of measure and state the meaning of the symbols they choose. *For example , when figuring out the area of a rectangle they record their answers in square units.* | Develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning.<br><br>Are careful about specifying units of measure and state the meaning of the symbols they choose. *For instance, they use appropriate labels when creating a line plot.* | Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning.<br><br>Use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids.<br><br>Are careful about specifying units of measure and state the meaning of the symbols they choose. *For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.* |
| | | | |
| | | | |

| Mathematical Practices | 3 | 4 | 5 |
|---|---|---|---|
| **7. Look for and make use of structure. (Deductive Reasoning)** | Look closely to discover a pattern or structure. *For example, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).* | Look closely to discover a pattern or structure. *For instance, students use properties of operations to explain calculations (partial products model).*<br><br>Relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting.<br><br>Generate number or shape patterns that follow a given rule. | Look closely to discover a pattern or structure. *For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals.*<br><br>Examine numerical patterns and relate them to a rule or a graphical representation. |
| **8. Look for and express regularity in repeated reasoning. (Inductive Reasoning)** | Notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. *For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56.*<br><br>Continually evaluate their work by asking themselves, "Does this make sense?" | Notice repetitive actions in computation to make generalizations<br><br>Use models to explain calculations and understand how algorithms work.<br><br>Use models to examine patterns and generate their own algorithms. *For example, students use visual fraction models to write equivalent fractions.* | Use repeated reasoning to understand algorithms and make generalizations about patterns.<br><br>Connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths.<br><br>Explore operations with fractions with visual models and begin to formulate generalizations. |

## AERO Mathematics Standards
## Updated 7/1/2015

| Progressions 6-8 | | | |
|---|---|---|---|
| **Domain: Ratios and Proportional Relationships** | **6** | **7** | **8** |
| Understand ratio concepts and use ratio reasoning to solve problems. | AERO. 6.RP.1 **DOK 1,2**<br>Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities | AERO. 7.RP.2 **DOK 1,2**<br>Recognize and represent proportional relationships between quantities | |
| | AERO. 6.RP.2 **DOK 1,2**<br>Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship | AERO. 7.RP.1 **DOK 1,2**<br>Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. | |
| | AERO. AERO. 6.RP.3 **DOK 1,2**<br>Use ratio and rate reasoning to solve real-world and mathematical problems | AERO. AERO. 7.RP.2a **DOK 1,2**<br>Decide whether two quantities are in a proportional relationship | |
| | AERO. 6.RP.3a **DOK 1,2**<br>Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. | AERO. 7.RP.2d **DOK 1,2**<br>Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. | AERO. 8.EE.5 **DOK 1,2,3**<br>Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. |

| **Domain: Ratios and Proportional Relationships** | **6** | **7** | **8** |
|---|---|---|---|
| Understand ratio concepts and use ratio reasoning to solve problems. | | AERO. 7.RP.2b **DOK 1,2**<br>Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships | AERO. 8.EE.6 **DOK 1,2,3**<br>Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b |
| | AERO. 6.RP.3b **DOK 1,2**<br>Solve unit rate problems including those involving unit pricing and constant speed. | AERO. 7.RP.2c **DOK 1,2**<br>Represent proportional relationships by equations. | |
| | AERO. 6.RP.3c **DOK 1,2**<br>Find a percent of a quantity as a rate per 100 ; solve problems involving finding the whole, given a part and the percent | | |
| | AERO. 6.RP.3d **DOK 1,2**<br>Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. | AERO. 7.RP.3 **DOK 1,2**<br>Use proportional relationships to solve multistep ratio and percent problems.. | |

| **Domain: The Number System** | **6** | **7** | **8** |
| --- | --- | --- | --- |
| **Apply and extend previous understandings of multiplication and division to divide fractions by fractions** | AERO. 6.NS.1 **DOK 1,2**<br>Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.? | AERO. 7.NS.1 **DOK 1,2**<br>Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; | |
| | | AERO. 7.NS.1a **DOK 1,2**<br>Describe situations in which opposite quantities combine to make 0 | |
| | | AERO. 7.NS.1b **DOK 1,2**<br>Understand p + q as the number located a distance \|q\| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. | |
| | | AERO. 7.NS.1c **DOK 1,2**<br>Understand subtraction of rational numbers as adding the additive inverse, p - q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts | |
| | | AERO. 7.NS.1d **DOK 1,2**<br>Apply properties of operations as strategies to add and subtract rational numbers. | |
| | | AERO. 7.NS.2c **DOK 1,2**<br>Apply properties of operations as strategies to add and subtract rational numbers | |

| Domain: The Number System | 6 | 7 | 8 |
| --- | --- | --- | --- |
| **Compute fluently with multi-digit numbers and find common factors and multiples.** | AERO. 6.NS.2 **DOK 1**<br>Fluently divide multi-digit numbers using the standard algorithm. | AERO. 7.NS.2d **DOK 1,2**<br>Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. | |
| **Know that there are numbers that are not rational, and approximate them by rational numbers.** | | | AERO. 8.NS.1 **DOK 1**<br>Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. |
| **Compute fluently with multi-digit numbers and find common factors and multiples.** | AERO. 6.NS.3 **DOK 1**<br>Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. | AERO. 7.NS.3 **DOK 1,2**<br>Solve real-world and mathematical problems involving the four operations with rational numbers | |
| | AERO. 6.NS.4 **DOK 1**<br>Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.<br><br>Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. | | |

| Domain: The Number System | 6 | 7 | 8 |
|---|---|---|---|
| **Apply and extend previous understandings of numbers to the system of rational numbers** | AERO. 6.NS.5 **DOK 1,2**<br>Understand that positive and negative numbers are used together to describe quantities having opposite directions or values ; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation | AERO. 7.NS.2 **DOK 1,2**<br>Describe situations in which opposite quantities combine to make 0. | |
| | AERO. 6.NS.6 **DOK 1**<br>Understand a rational number as a point on the number line.<br><br>Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates | | |
| | AERO. 6.NS.6a **DOK 1**<br>Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. | AERO. 7.NS.2a **DOK 1,2**<br>Understand p + q as the number located a distance \|q\| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. | AERO. 8.NS.2 **DOK 1,2**<br>Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2). |
| | | AERO. 7.NS.2b **DOK 1,2**<br>Understand subtraction of rational numbers as adding the additive inverse, p - q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. | |

| **Domain: The Number System** | **6** | **7** | **8** |
|---|---|---|---|
| **Apply and extend previous understandings of numbers to the system of rational numbers** | AERO.6.NS.6b **DOK 1**<br>Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes | | |
| | AERO. 6.NS.6c **DOK 1**<br>Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane | | |
| | AERO. 6.NS.7 **DOK 1,2**<br>Understand ordering and absolute value of rational numbers. | | |
| | AERO. 6.NS.7a **DOK 1,2**<br>Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. | | |
| | AERO. 6.NS.7b **DOK 1,2**<br>Write, interpret, and explain statements of order for rational numbers in real-world contexts.. | | |
| | AERO. 6.NS.7c **DOK 1,2**<br>Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. | | |

| **Domain: The Number System** | **6** | **7** | **8** |
|---|---|---|---|
| **Apply and extend previous understandings of numbers to the system of rational numbers** | AERO. 6.NS.7d **DOK 1,2**<br>Distinguish comparisons of absolute value from statements about order. | | |
| | AERO. 6.NS.8 **DOK 1,2**<br>Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. | | |

| Domain: Expressions and Equations | 6 | 7 | 8 |
|---|---|---|---|
| **Apply and extend previous understandings of arithmetic to algebraic expressions.**<br><br>**Work with radicals and integer exponents (G.8)** | AERO. 6.EE.1 **DOK 1**<br>Write and evaluate numerical expressions involving whole-number exponents | | AERO. 8.EE.1 **DOK 1**<br>Know and apply the properties of integer exponents to generate equivalent numerical expressions. |
| | AERO. 6.EE.2 **DOK 1,2**<br>Write, read, and evaluate expressions in which letters stand for numbers. | | AERO. 8.EE.2 **DOK 1**<br>Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number.<br><br>Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational |
| | AERO. 6.EE.3 **DOK 1,2**<br>Write expressions that record operations with numbers and with letters standing for numbers. | | |
| | AERO. 6.EE.3a **DOK 1,2**<br>Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. | | |
| | AERO. 6.EE.3b **DOK 1,2**<br>Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems.<br>Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). | AERO. 7.EE.2 **DOK 1,2**<br>Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. | AERO. 8.EE.3 **DOK 1,2**<br>Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. |

| Domain: Expressions and Equations | 6 | 7 | 8 |
|---|---|---|---|
| **Apply and extend previous understandings of arithmetic to algebraic expressions.**<br><br>**Work with radicals and integer exponents (G.8)** | AERO. 6.EE.3c **DOK 1,2**<br>Apply the properties of operations to generate equivalent expressions. | AERO. 7.EE.1 **DOK 1**<br>Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. | AERO. 8.EE.4 **DOK 1,2**<br>Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.<br>Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities Interpret scientific notation that has been generated by technology |
| | AERO. 6.EE.4 **DOK 1**<br>Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). | | |
| | AERO. 6.EE.5 **DOK 1**<br>Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true?<br><br>Use substitution to determine whether a given number in a specified set makes an equation or inequality true. | | |
| | AERO. 6.EE.6 **DOK 1,2**<br>Use variables to represent numbers and write expressions when solving a real-world or mathematical problem;<br><br>understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set | | |

| **Domain: Expressions and Equations** | **6** | **7** | **8** |
|---|---|---|---|
| **Apply and extend previous understandings of arithmetic to algebraic expressions.**<br><br>**Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (Grade 7)** | AERO. 6.EE.7 **DOK 1,2**<br>Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers. | AERO. 7.EE.3 **DOK 1,2,3**<br>Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.<br><br>Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. | |
| | AERO. 6.EE.8 **DOK 1,2**<br>Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams. | | |
| | AERO. 6.EE.9 **DOK 1,2,3**<br>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. | AERO. 7.EE.4 **DOK 1,2,3**<br>Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities | |

| **Domain: Expressions and Equations** | **6** | **7** | **8** |
|---|---|---|---|
| **Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (Grade 7)** | | AERO. 7.EE.4a **DOK 1,2,3**<br>Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers.<br><br>Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. | |
| | | AERO. 7.EE.4b **DOK 1,2,3**<br>Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers.<br><br>Graph the solution set of the inequality and interpret it in the context of the problem. | |
| **Analyze and solve linear equations and pairs of simultaneous linear equations.** | | | AERO. 8.EE.7 **DOK 1,2**<br>Solve linear equations in one variable. |
| | | | AERO. 8.EE.7a **DOK 1,2**<br>Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). |

| **Domain: Expressions and Equations** | **6** | **7** | **8** |
|---|---|---|---|
| **Analyze and solve linear equations and pairs of simultaneous linear equations.** | | | AERO. 8.EE.7b **DOK 1,2**<br>Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. |
| | | | AERO. 8.EE.8 **DOK 1,2,3**<br>Analyze and solve pairs of simultaneous linear equations. |
| | | | AERO. 8.EE.8a **DOK 1,2,3**<br>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. |
| | | | AERO. 8.EE.8b **DOK 1,2,3**<br>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. |
| | | | AERO. 8.EE.8c **DOK 1,2,3**<br>Solve real-world and mathematical problems leading to two linear equations in two variables. |

| **Domain: Geometry** | **6** | **7** | **8** |
|---|---|---|---|
| **Solve real-world and mathematical problems involving area, surface area, and volume.** | AERO. 6.G.1 **DOK 1,2**<br>Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes;<br><br>apply these techniques in the context of solving real-world and mathematical problems | AERO. 7.G.4 **DOK 1,2**<br>Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle | AERO. 8.G.9 **DOK 1,2**<br>Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. |
| | AERO. 6.G.2 **DOK 1,2**<br>Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism.<br><br>Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems | AERO. 7.G.6 **DOK 1,2**<br>Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. | |
| | AERO. 6.G.3 **DOK 1,2**<br>Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate.<br><br>Apply these techniques in the context of solving real-world and mathematical problem | | |

| **Domain: Geometry** | **6** | **7** | **8** |
|---|---|---|---|
| **Draw construct, and describe geometrical figures and describe the relationships between them.** | AERO. 6.G.4 **DOK 1,2**<br>Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.<br><br>Apply these techniques in the context of solving real-world and mathematical problems. | AERO. 7.G.1 **DOK 1,2**<br>Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. | |
| | | AERO. 7.G.2 **DOK 1,2**<br>Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. | |
| | | AERO. 7.G.3 **DOK 1,2**<br>Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. | |
| | | AERO. 7.G.5 **DOK 1,2**<br>Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure | |

| **Domain: Geometry** | **6** | **7** | **8** |
|---|---|---|---|
| **Understand congruence and similarity using physical models, transparencies, or geometry software.** | | | AERO. 8.G.1 **DOK 2**<br>Verify experimentally the properties of rotations, reflections, and translations: |
| | | | AERO. 8.G.1a **DOK 2**<br>Lines are taken to lines, and line segments to line segments of the same length.\ |
| | | | AERO. 8.G.1b **DOK 2**<br>Angles are taken to angles of the same measure. |
| | | | AERO. 8.G.1c **DOK 2**<br>Parallel lines are taken to parallel lines. |
| | | | AERO. 8.G.2 **DOK 1,2**<br>Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. |
| | | | AERO. 8.G.3 **DOK 1,2**<br>Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates |

| **Domain: Geometry** | **6** | **7** | **8** |
|---|---|---|---|
| Understand congruence and similarity using physical models, transparencies, or geometry software. | | | AERO. 8.G.4 **DOK 1,2**<br>Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. |
| | | | AERO. 8.G.5 **DOK 1,2,3**<br>Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. |
| Understand and apply the Pythagorean Theorem. | | | AERO. 8.G.6 **DOK 2,3**<br>Explain a proof of the Pythagorean Theorem and its converse. |
| | | | AERO. 8.G.7 **DOK 1,2**<br>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. |
| | | | AERO. 8.G.8 **DOK 1,2**<br>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system |

| **Domain: Functions** | **6** | **7** | **8** |
|---|---|---|---|
| Define, evaluate, and compare functions. | | | AERO. 8.F.1 **DOK 1,2**<br>Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output |
| | | | AERO. 8.F.2 **DOK 1,2**<br>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| | | | AERO. 8.F.3 **DOK 1,2**<br>Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. |

| **Domain: Functions** | **6** | **7** | **8** |
|---|---|---|---|
| Use functions to model relationships between quantities. | | | AERO. 8.F.4 **DOK 1,2,3**<br>Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values |
| | | | AERO. 8.F.5 **DOK 1,2,3**<br>Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).Sketch a graph that exhibits the qualitative features of a function that has been described verbally |

| **Domain: Statistics and Probability** | **6** | **7** | **8** |
|---|---|---|---|
| Develop understanding of statistical variability.<br><br>Use random sampling to draw inferences about a population (Grade 3) | AERO. 6.SP.1 **DOK 1**<br>Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. | AERO. 7.SP.1 **DOK 2**<br>Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.<br><br>Understand that random sampling tends to produce representative samples and support valid inferences. | |
| | AERO. 6.SP.2 **DOK 1,2**<br>Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. | AERO. 7.SP.2 **DOK 2,3**<br>Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. | |
| | AERO. 6.SP.3 **DOK 1**<br>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number | | |
| Draw informal comparative inferences about two populations. (Grade 7) | | AERO. 7.SP.3 **DOK 2,3**<br>Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. | |

| Domain: Statistics and Probability | 6 | 7 | 8 |
|---|---|---|---|
| Summarize and describe distributions.<br><br>Investigate patterns of association in bivariate data.(Grade 8) | AERO. 6.SP.4 **DOK 1,2**<br>Display numerical data in plots on a number line, including dot plots, histograms, and box plots | | AERO. 8.SP.1 **DOK 1,2,3**<br>Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association |
| | AERO. 6.SP.5 **DOK 1,2,3**<br>Summarize numerical data sets in relation to their context, such as by:<br>Reporting the number of observations | | AERO. 8.SP.2 **DOK 1,2**<br>Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. |
| | AERO. 6.SP.5b **DOK 1,2,3**<br>Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. | | AERO. 8.SP.3 **DOK 1,2**<br>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. |
| | AERO. 6.SP.5c **DOK 1,2,3**<br>Use quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. | | AERO. 8.SP.4 **DOK 1,2,3**<br>Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.<br><br>Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. |

| Domain: Statistics and Probability | 6 | 7 | 8 |
|---|---|---|---|
| **Summarize and describe distributions.**<br><br>**Draw informal comparative inferences about two populations. (Grade 7)** | AERO. 6.SP.5d **DOK 1,2,3**<br>Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. | AERO. 7.SP.4 **DOK 2,3**<br>Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. | |
| **Investigate chance processes and develop, use, and evaluate probability models.** | | AERO. 7.SP.5 **DOK 1**<br>Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. | |
| | | AERO. 7.SP.6 **DOK 2,3**<br>Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.. | |
| | | AERO. 7.SP.7 **DOK 2,3**<br>Develop a probability model and use it to find probabilities of events.<br><br>Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. | |

| **Domain: Statistics and Probability** | **6** | **7** | **8** |
|---|---|---|---|
| **Investigate chance processes and develop, use, and evaluate probability models.** | | AERO. 7.SP.7a **DOK 2,3**<br>Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. | |
| | | AERO. 7.SP.7b **DOK 2,3**<br>Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. | |
| | | AERO. 7.SP.8a **DOK 1, 2,3**<br>Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs | |
| | | AERO. 7.SP.8b **DOK 1, 2,3**<br>Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event | |
| | | AERO. 7.SP.8c **DOK 1,2,3**<br>Design and use a simulation to generate frequencies for compound events. | |

| Mathematical Practices | 6 | 7 | 8 |
|---|---|---|---|
| 1. Make sense of problems and persevere in solving them. | Solve problems involving ratios and rates and discuss how they solved them.<br><br>Solve real world problems through the application of algebraic and geometric concepts.<br><br>Seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". | Solve problems involving ratios and rates and discuss how they solved them.<br><br>Solve real world problems through the application of algebraic and geometric concepts.<br><br>Seek the meaning of a problem and look for efficient ways to represent and solve it.<br><br>Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?". | Solve real world problems through the application of algebraic and geometric concepts.<br><br>Seek the meaning of a problem and look for efficient ways to represent and solve it.<br><br>Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?" |
| 2. Reason abstractly and quantitatively. | Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities.<br><br>Contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. | Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities.<br><br>Contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. | Represent a wide variety of real world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities.<br><br>Examine patterns in data and assess the degree of linearity of functions.<br><br>Contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations. |

| Mathematical Practices | 6 | 7 | 8 |
| --- | --- | --- | --- |
| 3. Construct viable arguments and critique the reasoning of others. | Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.).<br><br>Refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students.<br><br>Pose questions like "How did you get that?", "Why is that true?" "Does that always work?"<br><br>Explain their thinking to others and respond to others' thinking. | Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.).<br><br>Refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students.<br><br>Pose questions like "How did you get that?", "Why is that true?" "Does that always work?". They explain their thinking to others and respond to others' thinking. | Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.).<br><br>Refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students.<br><br>Pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking. |

| Mathematical Practices | 6 | 7 | 8 |
|---|---|---|---|
| 4. Model with mathematics. | Model problem situations symbolically, graphically, tabularly, and contextually.<br><br>Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations.<br><br>Explore covariance and represent two quantities simultaneously.<br><br>Use number lines to compare numbers and represent inequalities.<br><br>Use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences about and make comparisons between data sets.<br><br>Connect and explain the connections between the different representations.<br><br>Use all of these representations as appropriate to a problem context. | Model problem situations symbolically, graphically, tabularly, and contextually.<br><br>Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations.<br><br>Explore covariance and represent two quantities simultaneously.<br><br>Use measures of center and variability and data displays (i.e. box plots and histograms) to draw inferences, make comparisons and formulate predictions. Students use experiments or simulations to generate data sets and create probability models.<br><br>Connect and explain the connections between the different representations.<br><br>Use all of these representations as appropriate to a problem context. | Model problem situations symbolically, graphically, tabularly, and contextually.<br><br>Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations.<br><br>Solve systems of linear equations and compare properties of functions provided in different forms.<br><br>Use scatterplots to represent data and describe associations between variables.<br><br>Connect and explain the connections between the different representations.<br><br>Use all of these representations as appropriate to a problem context. |

| Mathematical Practices | 6 | 7 | 8 |
|---|---|---|---|
| 5. Use appropriate tools strategically. | Consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 6 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data.<br><br>Use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures. | Consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 7 may decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data.<br><br>Use physical objects or applets to generate probability data and use graphing calculators or spreadsheets to manage and represent data in different forms. | Consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set.<br><br>Draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal. |
| 6. Attend to precision. | Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning.<br><br>Use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations or inequalities. | Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning.<br><br>Define variables, specify units of measure, and label axes accurately.<br><br>Use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations or inequalities. | Continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning.<br><br>Use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. |

| Mathematical Practices | 6 | 7 | 8 |
| --- | --- | --- | --- |
| 7. Look for and make use of structure. (Deductive Reasoning) | Routinely seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties.<br><br>Apply properties to generate equivalent expressions (i.e. 6 + 2x = 2 (3 + x) by distributive property) and solve equations (i.e. 2c + 3 = 15, 2c = 12 by subtraction property of equality; c=6 by division property of equality).<br><br>Compose and decompose two- and three-dimensional figures to solve real world problems involving area and volume. | Seek patterns or structures to model and solve problems. For instance, students recognize patterns that exist in ratio tables making connections between the constant of proportionality in a table with the slope of a graph.<br><br>Apply properties to generate equivalent expressions (i.e. 6 + 2x = 2 (3 + x) by distributive property) and solve equations (i.e. 2c + 3 = 15, 2c = 12 by subtraction property of equality; c=6 by division property of equality).<br><br>Compose and decompose two- and three-dimensional figures to solve real world problems involving scale drawings, surface area, and volume.<br><br>Examine tree diagrams or systematic lists to determine the sample space for compound events and verify that they have listed all possibilities. | Seek patterns or structures to model and solve problems.<br><br>Apply properties to generate equivalent expressions and solve equations.<br><br>Examine patterns in tables and graphs to generate equations and describe relationships.<br><br>Experimentally verify the effects of transformations and describe them in terms of congruence and similarity. |

| Mathematical Practices | 6 | 7 | 8 |
|---|---|---|---|
| 8. Look for and express regularity in repeated reasoning. (Inductive Reasoning) | Use repeated reasoning to understand algorithms and make generalizations about patterns.<br><br>Solve and model problems, noticing that a/b ÷ c/d = ad/bc and construct other examples and models that confirm their generalization.<br><br>Connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals.<br><br>Informally begin to make connections between covariance, rates, and representations showing the relationships between quantities. | Use repeated reasoning to understand algorithms and make generalizations about patterns.<br><br>Solve and model problems, noticing that a/b ÷ c/d = ad/bc and construct other examples and models that confirm their generalization.<br><br>Extend their thinking to include complex fractions and rational numbers.<br><br>Formally begin to make connections between covariance, rates, and representations showing the relationships between quantities.<br><br>Create, explain, evaluate, and modify probability models to describe simple and compound events. | Use repeated reasoning to understand algorithms and make generalizations about patterns.<br><br>Use iterative processes to determine more precise rational approximations for irrational numbers.<br><br>Analyze patterns of repeating decimals to identify the corresponding fraction.<br><br>Solve and model problems, noticing that the slope of a line and rate of change are the same value.<br><br>Flexibly make connections between covariance, rates, and representations showing the relationships between quantities. |

# AERO Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example: (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

## HS Conceptual Category: Number and Quantity

**Numbers and Number Systems.** During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers .With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings. Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5. Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

**Quantities.** In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

| **Domains** | **The Real Number System HSN-RN** | **Quantities★ HSN -Q** | **The Complex Number System HSN - CN** | **Vector and Matrix Quantities HSN –VM** |
|---|---|---|---|---|
| **Clusters** | Extend the properties of exponents to rational exponents<br><br>Use properties of rational and irrational numbers. | Reason quantitatively and use units to solve problems | Perform arithmetic operations with complex Numbers<br><br>Represent complex numbers and their operations on the complex plane<br><br>Use complex numbers in polynomial identities and equations | Represent and model with vector quantities.<br><br>Perform operations on vectors.<br><br>Perform operations on matrices and use matrices in applications. |

| Domains | The Real Number System HSN-RN | Quantities★ HSN -Q | The Complex Number System HSN -CN | Vector and Matrix Quantities HSN - VM |
|---|---|---|---|---|
| **Clusters/**<br><br>**Standards** | **Extend the properties of exponents to rational exponents**<br><br>**AERO**.HSN-RN.1 **DOK 1,2**<br>Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.* | **Reason quantitatively and use units to solve problems**<br><br>**AERO** HSN-Q. 1. **DOK 1,2**<br>Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | **Perform arithmetic operations with complex numbers**<br><br>**AERO** HSN.CN.1 **DOK 1**<br>1. Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form *a* + *bi* with *a* and *b* real. | **Represent and model with vector quantities**<br><br>**AERO**. HSN.VM. 1. (+) **DOK 1**<br>Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v*, \|*v*\|, \|\|*v*\|\|, *v*). |
| | | **AERO** HSN-Q. 2 **DOK 1,2**<br>Define appropriate quantities for the purpose of descriptive modeling.<br>. | **AERO** HSN.CN.2 **DOK 1**<br>Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. | **AERO**. HSN.VM. 2. (+) **DOK 1**<br>Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. |
| | | **AERO** HSN-Q. 3 **DOK 1,2**<br>Choose a level of accuracy appropriate to limitations on measurement when reporting quantities | **AERO**. HSN.CN. 3. (+) **DOK 1**<br>Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | **AERO**. HSN.VM. 3. (+) **DOK 1,2**<br>Solve problems involving velocity and other quantities that can be represented by vectors. |

| Domains | The Real Number System HSN-RN | Quantities★ HSN -Q | The Complex Number System HSN - CN | Vector and Matrix Quantities HSN - VM |
|---|---|---|---|---|
| **Clusters/**<br><br>**Standards** | **Extend the properties of exponents to rational exponents**<br><br>**AERO. HSN-RN**.2 **DOK 1**<br>Rewrite expressions involving radicals and rational exponents using the properties of exponents. Use properties of rational and irrational numbers. | | **Represent complex numbers and their operations on the complex plane**.<br><br>**AERO.HSN.CN.4. (+) DOK 1,2**<br>Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | **Perform operations on vectors.**<br><br>**AERO. HSN.VM. 4. (+) DOK 1,2**<br>Add and subtract vectors.<br>a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.<br><br>b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.<br><br>c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of $w$, with the same magnitude as $w$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. |
| | **AERO.HSN-RN.3 DOK 1,2**<br>Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | | **AERO.HSN.CN.5. (+) DOK 1,2**<br>Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus* 2 *and argument* 120°. | **AERO. HSN.VM. 5. (+) DOK 1,2**<br>Multiply a vector by a scalar.<br>a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise<br><br>b. Compute the magnitude of a scalar multiple $cv$ using $\|\|cv\|\| = \|c\|v$. Compute the direction of $cv$ knowing that when $\|c\|v \neq 0$, the direction of $cv$ is either along $v$ (for $c > 0$) or against $v$ (for $c < 0$). |
| | | | **AERO.HSN.CN.6. (+) DOK 1**<br><br>Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | |

| **Domains** | **The Real Number System HSN-RN** | Quantities★ HSN -Q | **The Complex Number System HSN -CN** | **Vector and Matrix Quantities HSN -VM** |
|---|---|---|---|---|
| **Clusters/**<br><br>**Standards** | | | **Use complex numbers in polynomial identities and equations.**<br><br>**AERO.HSN.CN.7**. DOK 1<br>Solve quadratic equations with real coefficients that have complex solutions. | **Perform operations on matrices and use matrices in applications**<br><br>**AERO. HSN.VM. 6. (+)** DOK 1,2<br>Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. |
| | | | **AERO.HSN.CN.8. (+)** DOK 1,2<br>Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$. | **AERO. HSN.VM. 7. (+)** DOK 1<br>Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.<br><br>. |
| | | | **AERO.HSN.CN.9. (+)** DOK 1,2<br>Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | **AERO. HSN.VM. 8. (+)** DOK 1<br>Add, subtract, and multiply matrices of appropriate dimensions. |
| | | | | **AERO. HSN.VM. 9. (+)** DOK 1<br>Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. |
| | | | | **AERO. HSN.VM. 10. (+)** DOK 1<br>Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. |

| **Domains** | **The Real Number System HSN-RN** | Quantities★ HSN -Q | **The Complex Number System HSN -CN** | **Vector and Matrix Quantities HSN -VM** |
|---|---|---|---|---|
| **Clusters/**<br><br>**Standards** | | | | **AERO. HSN.VM. 11. (+)** **DOK 1,2**<br>Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. |
| | | | | **AERO. HSN.VM. 12. (+)** **DOK 1,2**<br>Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area |

## HS Conceptual Category: Algebra

**Expressions.** An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances. Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, p + 0.05p can be interpreted as the addition of a 5% tax to a price p. Rewriting p + 0.05p as 1.05pshows that adding a tax is the same as multiplying the price by a constant factor. Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure. A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

**Equations and inequalities**. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of x + 1 = 0 is an integer, not a whole number; the solution of 2x + 1 = 0 is a rational number, not an integer; the solutions of x2 – 2 = 0 are real numbers, not rational numbers; and the solutions of x2 + 2 = 0are complex numbers, not real numbers. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, A = ((b1+b2)/2)h, can be solved for h using the same deductive process. Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

**Connections to Functions and Modeling**. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

| **Domains** | **Seeing Structure in Expressions HSA.SSE** | **Arithmetic with Polynomials and Rational Expressions HSAAPR** | **Creating Equations HSA.CED** | **Reasoning with Equations and Inequalities HSA.REI** |
|---|---|---|---|---|
| **Clusters** | **Interpret the structure of expressions**<br><br>**Write expressions in equivalent forms to solve problems** | **Perform arithmetic operations on polynomials**<br><br>**Understand the relationship between zeros and factors of polynomials**<br><br>**Use polynomial identities to solve problems**<br><br>**Rewrite rational expressions** | **Create equations that describe numbers or relationships** | **Understand solving equations as a process of reasoning and explain the reasoning**<br><br>**Solve equations and inequalities in one variable**<br><br>**Solve systems of equations**<br><br>**Represent and solve equations and inequalities graphically** |

| Domains | Seeing Structure in Expressions HSA.SSE | Arithmetic with Polynomials and Rational Expressions HSAAPR | Creating Equations HSA.CED | Reasoning with Equations and Inequalities HSA.REI |
|---|---|---|---|---|
| **Clusters/ Standards** | **Interpret the structure of expressions**<br><br>**AERO.HSA.SSE.**1.* **DOK 1,2**<br>Interpret expressions that represent a quantity in terms of its context.<br><br>a. Interpret parts of an expression, such as terms, factors, and coefficients.<br>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. | **Perform arithmetic operations on polynomials**<br><br>**AERO.HSAAPR.1** **DOK 1**<br>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. | **Create equations that describe numbers or relationships**<br><br>**AERO.HSA.CED.**1. **DOK 1,2**<br>Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* | **Understand solving equations as a process of reasoning and explain the reasoning**<br><br>**AERO.HSA.REI.1** **DOK 1,2,3**<br>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| | **AERO.HSA.SSE.**2. **DOK 1,2**<br>Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. | | **AERO.HSA.CED.**2 **DOK 1,2**<br>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | **AERO.HSA.REI.2** **DOK 1,2**<br>Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. |
| | | | **AERO.HSA.CED.**3. **DOK 1,2,3**<br>Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. . | |
| | | | **AERO.HSA.CED.**4. **DOK 1**<br>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | |

| Domains | Seeing Structure in Expressions HSA.SSE | Arithmetic with Polynomials and Rational Expressions HSAAPR | Creating Equations HSA.CED | Reasoning with Equations and Inequalities HSA.REI |
|---|---|---|---|---|
| **Clusters/ Standards** | **Write expressions in equivalent forms to solve problems**<br><br>**AERO.HSA.SSE.3.*** **DOK 1,2**<br>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.<br><br>a. Factor a quadratic expression to reveal the zeros of the function it defines.<br>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.<br>c. Use the properties of exponents to transform expressions for exponential functions. | **Understand the relationship between zeros and factors of polynomials**<br><br>**AERO.HSAAPR.2** **DOK 1,2**<br>Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. | | **Solve equations and inequalities in one variable**<br><br>**AERO.HSA.REI.3** **DOK 1**<br>Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. |
| | **AERO.HSA.SSE.4** ***DOK 1,2,3**<br>Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* | **AERO.HSAAPR.3** **DOK 1,2**<br>Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial | | **AERO.HSA.REI.4** **DOK 1,2,3**<br>Solve quadratic equations in one variable.<br>a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)2 = q$ that has the same solutions. Derive the quadratic formula from this form.<br>b. Solve quadratic equations by inspection (e.g., for $x2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$. |

| Domains | Seeing Structure in Expressions HSA.SSE | Arithmetic with Polynomials and Rational Expressions HSAAPR | Creating Equations HSA.CED | Reasoning with Equations and Inequalities HSA.REI |
|---|---|---|---|---|
| **Clusters/ Standards** | | **Use polynomial identities to solve problems**<br><br>**AERO.HSAAPR.4** **DOK 1,2**<br>Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* | | **Solve systems of equations**<br><br>**AERO.HSA.REI.5** **DOK 2,3**<br>Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. |
| | | **AERO.HSAAPR.**5. (+) **DOK 1,2,3**<br>Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. | | **AERO.HSA.REI.6** **DOK 1,2**<br>Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. |
| | | | | **AERO.HSA.REI.7** **DOK 1,2**<br>Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$. |
| | | | | **AERO.HSA.REI.**8. (+) **DOK 1**<br>Represent a system of linear equations as a single matrix equation **i**n a vector variable. |
| | | | | **AERO.HSA.REI.**9. (+) **DOK 1,2**<br>Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). |

| Domains | Seeing Structure in Expressions HSA.SSE | Arithmetic with Polynomials and Rational Expressions HSAAPR | Creating Equations HSA.CED | Reasoning with Equations and Inequalities HSA.REI |
|---|---|---|---|---|
| **Clusters/ Standards** | | **Rewrite rational expressions**<br><br>**AERO**.**HSAAPR.**6. **DOK 1,2**<br>Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. | | **Represent and solve equations and inequalities graphically**<br><br>**AERO**.**HSA.REI**.10. **DOK 1**<br>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).<br><br>. |
| | | **AERO**.**HSAAPR.**7. (+) **DOK 1**<br>Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | | **AERO**.**HSA.REI.11.** **DOK 1,2,3**<br>Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. * |
| | | | | **AERO**.**HSA.REI.12**. **DOK 1,2**<br>Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes |

## HS Conceptual Category: Functions

**Functions** describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, $v$; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is $T$. The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties. Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships. A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

**Connections to Expressions, Equations, Modeling, and Coordinates.** Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

| Domains | Interpreting Functions<br>HSF.1F | Building Functions<br>HSF.BF | Linear, Quadratic, and Exponential Models<br>HSF.LE | Trigonometric Functions<br>HSF.TF |
|---|---|---|---|---|
| Clusters/ Standards | **Understand the concept of a function and use function notation**<br><br>**Interpret functions that arise in applications in terms of the context**<br><br>**Analyze functions using different representations** | **Build a function that models a relationship between two quantities**<br><br>**Build new functions from existing functions** | **Construct and compare linear, quadratic, and exponential models and solve problems**<br><br>**Interpret expressions for functions in terms of the situation they model** | **Extend the domain of trigonometric functions using the unit circle**<br><br>**Model periodic phenomena with trigonometric functions**<br><br>**Prove and apply trigonometric identities** |

| Domains | Interpreting Functions<br>HSF.1F | Building Functions<br>HSF.BF | Linear, Quadratic, and Exponential Models<br>HSF.LE | Trigonometric Functions<br>HSF.TF |
|---|---|---|---|---|
| Clusters/ Standards | **Understand the concept of a function and use function notation**<br><br>**AERO.HSF.1F.1** **DOK 1**<br>Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). | **Build a function that models a relationship between two quantities**<br><br>**AERO. HSF.BF.1** **DOK 1,2**<br>Write a function that describes a relationship between two quantities. *<br><br>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.<br><br>b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*<br><br>c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* | **Construct and compare linear, quadratic, and exponential models and solve problems**<br><br>**AERO.HSF.LE.1** **DOK 1,2.3**<br>Distinguish between situations that can be modeled with linear functions and with exponential functions.<br><br>a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.<br><br>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.<br><br>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | **Extend the domain of trigonometric functions using the unit circle**<br><br>**AERO.HSF.TF.1** **DOK 1**<br>Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. |
| | **AERO.HSF.1F.2** **DOK 1,2**<br>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | **AERO. HSF.BF.2** **DOK 1,2**<br>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. * | **AERO.HSF.LE.2** **DOK 1,2**<br>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).<br>context. | **AERO.HSF.TF.2** **DOK 1,2**<br>Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. |

| Domains | Interpreting Functions<br>HSF.1F | Building Functions<br>HSF.BF | Linear, Quadratic, and Exponential Models<br>HSF.LE | Trigonometric Functions<br>HSF.TF |
|---|---|---|---|---|
| Clusters/ Standards | **AERO.HSF.1F.3** **DOK 1**<br>Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$ | | **AERO.HSF.LE.3** **DOK 1,2**<br>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. | **AERO.HSF.TF.3. (+)** **DOK 1,2**<br>Use special triangles to determine geometrically the values of sine, cosine, tangent for ∏/3, ∏/4 and ∏/6, and use the unit circle to express the values of sine, cosine, and tangent for ∏–*x*, ∏+*x*, and 2∏–*x* in terms of their values for *x*, where *x* is any real number. |
| | | | **AERO.HSF.LE.4** **DOK 1**<br>For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. | **AERO.HSF.TF.4. (+)** **DOK 2**<br>Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. |
| | **Interpret functions that arise in applications in terms of the context**<br><br>**AERO.HSF.1F.4** **DOK 1,2**<br>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *<br>. | **Build new functions from existing functions**<br><br>**AERO. HSF.BF.3** **DOK 1,2**<br>Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | **Interpret expressions for functions in terms of the situation they model**<br><br>**AERO.HSF.LE.5** **DOK 1,2**<br>Interpret the parameters in a linear or exponential function in terms of a | **Model periodic phenomena with trigonometric functions**<br><br>**AERO.HSF.TF.5** **DOK 1,2**<br>Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. * |

| Domains | Interpreting Functions HSF.1F | Building Functions HSF.BF | Linear, Quadratic, and Exponential Models HSF.LE | Trigonometric Functions HSF.TF |
|---|---|---|---|---|
| **Clusters/ Standards** | **AERO.HSF.1F.5** **DOK 1,2**<br>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*★ | **AERO. HSF.BF.4** **DOK 1,2**<br>Find inverse functions.<br><br>a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.<br><br>b. (+) Verify by composition that one function is the inverse of another.<br><br>c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.<br><br>d. (+) Produce an invertible function from a non-invertible function by restricting the domain. | | **AERO.HSF.TF.6. (+)** **DOK 1,2**<br>Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. |
| | **AERO.HSF.1F.6** **DOK 1,2**<br>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph | **AERO. HSF.BF.5. (+)** **DOK 1,2**<br>Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | | **AERO.HSF.TF.7. (+)** **DOK 1,2,3**<br>Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. * |

| Domains | Interpreting Functions<br>HSF.1F | Building Functions<br>HSF.BF | Linear, Quadratic, and Exponential Models<br>HSF.LE | Trigonometric Functions<br>HSF.TF |
|---|---|---|---|---|
| **Clusters/ Standards** | **Analyze functions using different representations**<br><br>**AERO.HSF.1F.7** **DOK 1,2**<br>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★<br><br>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.<br><br>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.<br><br>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.<br><br>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.<br><br>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | | | **Prove and apply trigonometric identities**<br><br>**AERO.HSF.TF.8** **DOK 1,2,3**<br>Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle |

| Domains | Interpreting Functions<br>HSF.1F | Building Functions<br>HSF.BF | Linear, Quadratic, and Exponential Models<br>HSF.LE | Trigonometric Functions<br>HSF.TF |
|---|---|---|---|---|
| **Clusters/ Standards** | **AERO.HSF.1F.8** **DOK 1,2**<br>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.<br><br>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.<br><br>b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^{t}$ $y = (0.97)^{t}$ $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* | | | **AERO.HSF.TF.9 (+)** **DOK 1,2,3**<br>Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. |
| | **AERO.HSF.1F.9** **DOK 1,2**<br>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum* | | | |

# HS Conceptual Category: Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material. Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.) During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs.

Later in college some students develop Euclidean and other geometries carefully from a small set of axioms. The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures. Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent. The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to nonright triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-

Angle is not a congruence criterion. Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations. Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

**Connections to Equations.** The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

| Domains | Congruence<br>HSG.CO | Similarity, Right Triangles, and Trigonometry<br>HSG>SRT | Circles<br>HSG,CA | Expressing Geometric Properties with Equations<br>HSG.GPE | Geometric Measurement and Dimension<br>HSG.GMD | Modeling with Geometry<br>HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters** | **Experiment with transformations in the plane Understand congruence in terms of rigid motions**<br><br>**Prove geometric theorems**<br><br>**Make geometric constructions** | **Understand similarity in terms of similarity transformations**<br><br>**Prove theorems involving similarity**<br><br>**Define trigonometric ratios and solve problems involving right triangles**<br><br>**Apply trigonometry to general triangles** | **Understand and apply theorems about circles**<br><br>**Find arc lengths and areas of sectors of circles** | **Translate between the geometric description and the equation for a conic section**<br><br>**Use coordinates to prove simple geometric theorems algebraically** | **Explain volume formulas and use them to solve problems**<br><br>**Visualize relationships between two dimensional and three-dimensional objects** | **Apply geometric concepts in modeling situations** |

| Domains | Congruence<br>HSG.CO | Similarity, Right Triangles, and Trigonometry<br>HSG.SRT | Circles<br>HSG.C | Expressing Geometric Properties with Equations<br>HSG.GPE | Geometric Measurement and Dimension<br>HSG.GMD | Modeling with Geometry<br>HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters/ Standards** | Experiment with transformations in the plane<br><br>**AERO.HSG.CO.1**<br>**DOK 1**<br>Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | **Understand similarity in terms of similarity transformations**<br><br>**AERO.HSG.SRT.1**<br>**DOK 2**<br>Verify experimentally the properties of dilations given by a center and a scale factor:<br>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.<br>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | **Understand and apply theorems about circles**<br><br>**AERO.HSG.C.1**<br>**DOK 3**<br>Prove that all circles are similar. | **Translate between the geometric description and the equation for a conic section**<br><br>**AERO.HSG.GPE.1**<br>**DOK 1,2,3**<br>Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | **Explain volume formulas and use them to solve problems**<br><br>**AERO.HSG.GMD.1**<br>**DOK 2,3**<br>Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* | **Apply geometric concepts in modeling situations**<br><br>**AERO.HSG.MG.1**<br>**DOK 1,2**<br>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). |

| Domains | Congruence HSG.CO | Similarity, Right Triangles, and Trigonometry HSG.SRT | Circles HSG.C | Expressing Geometric Properties with Equations HSG.GPE | Geometric Measurement and Dimension HSG.GMD | Modeling with Geometry HSG.MG |
|---|---|---|---|---|---|---|
| Clusters/ Standards | **AERO.HSG.CO.2** **DOK 1.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not | **AERO.HSG.SRT.2** **DOK 1,2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | **AERO.HSG.C.2** **DOK 1,2** Identify and describe relationships among inscribed angles, radii,and chords. *Include the relationship between central, inscribed, and circumscribed angles; the radius intersects the circle.* | **AERO.HSG.GPE.2** **DOK 1,2** Derive the equation of a parabola given a focus and directrix. | **AERO.HSG.GMD.2 +** **DOK 2,3** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. | **AERO.HSG.MG.2** **DOK 1,2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★ |
| | **AERO.HSG.CO.3** **DOK 1.2** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | **AERO.HSG.SRT.3** **DOK 2.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | **AERO.HSG.C.3** **DOK 2,3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle | **AERO.HSG.GPE.3(+)** **DOK 1,2** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant | **AERO.HSG.GMD.3** **DOK 1,2** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★ | **AERO.HSG.MG.3** **DOK 2,3,4** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical |
| | **AERO.HSG.CO.4** **DOK 2** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | | **AERO.HSG.C.4 (+)** **DOK 2** Construct a tangent line from a point outside a given circle to the circle. | | | |

| Domains | Congruence<br>HSG.CO | Similarity, Right Triangles, and Trigonometry<br>HSG.SRT | Circles<br>HSG.C | Expressing Geometric Properties with Equations<br>HSG.GPE | Geometric Measurement and Dimension<br>HSG.GMD | Modeling with Geometry<br>HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters/ Standards** | **AERO.HSG.CO.5**<br>**DOK 1.2**<br>Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another | | | | | |
| | **Understand congruence in terms of rigid motions**<br><br>**AERO.HSG.CO.6**<br>**DOK 1.2**<br>Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | **Prove theorems involving similarity**<br><br>**AERO.HSG.SRT.4**<br>**DOK 3**<br>Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* | **Find arc lengths and areas of sectors of circles**<br><br>**AERO.HSG.C.5**<br>**DOK 1,2,3**<br>Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | **Use coordinates to prove simple geometric theorems algebraically**<br><br>**AERO.HSG.GPE.4**<br>**DOK 3**<br>Use coordinates to prove simple geometric theorems algebraically. *).* | **Visualize relationships between two-dimensional and three dimensional objects**<br><br>**AERO.HSG.GMD.4**<br>**DOK 1,2**<br>Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | |

| Domains | Congruence HSG.CO | Similarity, Right Triangles, and Trigonometry HSG.SRT | Circles HSG.C | Expressing Geometric Properties with Equations HSG.GPE | Geometric Measurement and Dimension HSG.GMD | Modeling with Geometry HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters/ Standards** | **AERO.HSG.CO.7** **DOK 2.3** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | **AERO.HSG.SRT.5** **DOK 1,2,3** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | | **AERO.HSG.GPE.5** **DOK 1,2** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | | |
| | **AERO.HSG.CO.8** **DOK 2,3** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions | | | **AERO.HSG.GPE.6** **DOK 1,2** Find the point on a directed line segment between two given points that partitions the segment in a given ratio | | |
| | | | | **AERO.HSG.GPE.7** **DOK 1,2** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★ | | |

| Domains | Congruence HSG.CO | Similarity, Right Triangles, and Trigonometry HSG.SRT | Circles HSG.C | Expressing Geometric Properties with Equations HSG.GPE | Geometric Measurement and Dimension HSG.GMD | Modeling with Geometry HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters/ Standards** | **Prove geometric theorems**<br>**AERO.HSG.CO.9**<br>**DOK 3**<br>Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* | **Define trigonometric ratios and solve problems involving right triangles**<br><br>**AERO.HSG.SRT.6**<br>**DOK 1,2**<br>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | | | | |
| | **AERO.HSG.CO.10**<br>**DOK 3**<br>Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* | **AERO.HSG.SRT.7**<br>**DOK 1,2**<br>Explain and use the relationship between the sine and cosine of complementary angles. | | | | |

| Domains | Congruence HSG.CO | Similarity, Right Triangles, and Trigonometry HSG.SRT | Circles HSG.C | Expressing Geometric Properties with Equations HSG.GPE | Geometric Measurement and Dimension HSG.GMD | Modeling with Geometry HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters/ Standards** | **AERO.HSG.CO.11** **DOK 3** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals* | **AERO.HSG.SRT.8** **DOK 1,2** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. * | | | | |
| | **Make geometric constructions** <br><br> **AERO.HSG.CO.12** **DOK 2** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). . | **Apply trigonometry to general triangles** <br><br> **AERO.HSG.SRT.9.** **DOK 2,3** **(+)** Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | | | | |
| | **AERO.HSG.CO.13** **DOK 2** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | **AERO.HSG.SRT.10.**(+) **DOK 1,2,3** Prove the Laws of Sines and Cosines and use them to solve problems. | | | | |

| Domains | Congruence HSG.CO | Similarity, Right Triangles, and Trigonometry HSG.SRT | Circles HSG.C | Expressing Geometric Properties with Equations HSG.GPE | Geometric Measurement and Dimension HSG.GMD | Modeling with Geometry HSG.MG |
|---|---|---|---|---|---|---|
| **Clusters/ Standards** | | **AERO.HSG.SRT.11(+)** **DOK 1,2** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g.,surveying problems, resultant forces). | | | | |

## HS Conceptual Category: Statistics and Probability

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account. Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range).Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken. Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn. Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables. Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

**Connections to Functions and Modeling**. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

| **Domains** | **Interpreting Categorical and Quantitative Data HSS.ID** | **Making Inferences and Justifying Conclusions HSS.IC** | **Conditional Probability and the Rules of Probability HSS.CP** | **Using Probability to Make Decisions HSS.MD** |
|---|---|---|---|---|
| **Clusters** | **Summarize, represent, and interpret data on a single count or measurement variable**<br><br>**Summarize, represent, and interpret data on two categorical and quantitative variables**<br><br>**Interpret linear models** | **Understand and evaluate random processes underlying statistical experiments**<br><br>**Make inferences and justify conclusions from sample surveys, experiments and observational studies** | **Understand independence and conditional probability and use them to interpret data**<br><br>**Use the rules of probability to compute probabilities of compound events in a uniform probability model** | **Calculate expected values and use them to solve problems**<br><br>**Use probability to evaluate outcomes of decisions** |

| Domains | Interpreting Categorical and Quantitative Data<br>HSS.ID | Making Inferences and Justifying Conclusions<br>HSS.IC | Conditional Probability and the Rules of Probability<br>HSS.CP | Using Probability to Make Decisions<br>HSS.MD |
|---|---|---|---|---|
| **Clusters/ Standards** | **Summarize, represent, and interpret data on a single count or measurement variable**<br><br>**AERO.HSS.ID.1** **DOK 1,2**<br>Represent data with plots on the real number line (dot plots, histograms, and box plots). | **Understand and evaluate random processes underlying statistical experiments**<br><br>**AERO.HSS.IC.1** **DOK 1**<br>Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | **Understand independence and conditional probability and use them to interpret data**<br><br>**AERO.HSS.CP.1** **DOK 1,2**<br>Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). | **Calculate expected values and use them to solve problems**<br><br>**AERO.HSS.MD.1. (+)** **DOK 1,2**<br>Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. |
| | **AERO.HSS.ID.2** **DOK 1,2**<br>Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | **AERO.HSS.IC.2** **DOK 1,2**<br>Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model* | **AERO.HSS.CP.2** **DOK 1**<br>Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | **AERO.HSS.MD.2. (+)** **DOK 1,2**<br>Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. |
| | **AERO.HSS.ID.3** **DOK 1,2**<br>Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | | **AERO.HSS.CP.3** **DOK 1,2**<br>Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. | **AERO.HSS.MD.3. (+)** **DOK 1,2,3**<br>Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. |

| Domains | Interpreting Categorical and Quantitative Data<br>HSS.ID | Making Inferences and Justifying Conclusions<br>HSS.IC | Conditional Probability and the Rules of Probability<br>HSS.CP | Using Probability to Make Decisions<br>HSS.MD |
|---|---|---|---|---|
| **Clusters/ Standards** | **AERO.HSS.ID.4** **DOK 1,2**<br>Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | | **AERO.HSS.CP.4** **DOK 1,2**<br>Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* | **AERO.HSS.MD.4. (+)** **DOK 1,2,3**<br>Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. |
| | | | **AERO.HSS.CP.5** **DOK 1,2,3**<br>Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* | |

| Domains | Interpreting Categorical and Quantitative Data<br>HSS.ID | Making Inferences and Justifying Conclusions<br>HSS.IC | Conditional Probability and the Rules of Probability<br>HSS.CP | Using Probability to Make Decisions<br>HSS.MD |
|---|---|---|---|---|
| **Clusters/ Standards** | **Summarize, represent, and interpret data on two categorical and quantitative variables**<br><br>**AERO.HSS.ID.5** DOK 1.2<br>Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). | **Make inferences and justify conclusions from sample surveys, experiments, and observational studies**<br><br>**AERO.HSS.IC.3** DOK 1,2<br>Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | **Use the rules of probability to compute probabilities of compound events in a uniform probability model**<br><br>**AERO.HSS.CP.6** DOK 1,2<br>Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model. | **Use probability to evaluate outcomes of decisions**<br><br>**AERO.HSS.MD.5. (+)** DOK 1,2,3<br>Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.<br><br>a. Find the expected payoff for a game of chance.<br><br>b. Evaluate and compare strategies on the basis of expected values. |
| | **AERO.HSS.ID.6** DOK 1,2<br>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br><br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*<br><br>b. Informally assess the fit of a function by plotting and analyzing residuals.<br><br>c. Fit a linear function for a scatter plot that suggests a linear association. | **AERO.HSS.IC.4** DOK 2<br>Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | **AERO.HSS.CP.7** DOK 1,2<br>Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. | **AERO.HSS.MD.6. (+)** DOK 1,2<br>Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). |

| Domains | Interpreting Categorical and Quantitative Data HSS.ID | Making Inferences and Justifying Conclusions HSS.IC | Conditional Probability and the Rules of Probability HSS.CP | Using Probability to Make Decisions HSS.MD |
|---|---|---|---|---|
| **Clusters/ Standards** | | **AERO.HSS.IC.5** **DOK 2,3**<br>Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. | **AERO.HSS.CP.8. (+)** **DOK 1,2**<br>Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B\|A) = P(B)P(A\|B), and interpret the answer in terms of the model. | **AERO.HSS.MD.7. (+)** **DOK 2,3**<br>Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). |
| | | **AERO.HSS.IC.6** **DOK 2.3**<br>Evaluate reports based on data. | **AERO.HSS.CP.9. (+)** **DOK 1,2**<br>Use permutations and combinations to compute probabilities of compound events and solve problems. | |
| | **Interpret linear models**<br><br>**AERO.HSS.ID.7** **DOK 1,2**<br>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | | | |
| | **AERO.HSS.ID.8** **DOK 1,2**<br>Compute (using technology) and interpret the correlation coefficient of a linear fit. | | | |
| | **AERO.HSS.ID.9** **DOK 1,2**<br>Distinguish between correlation and causation. | | | |

**High School—Modeling**

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

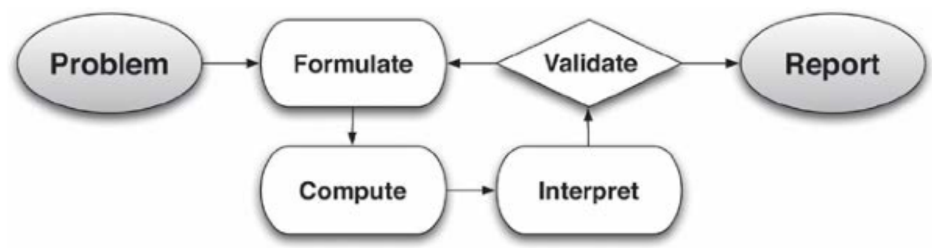
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

The basic modeling cycle is summarized as involving
(1) identifying variables in the situation and selecting those that represent essential features,

(2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
(3) analyzing and performing operations on these relationships to draw conclusions,
(4) interpreting the results of the mathematics in terms of the original situation,
(5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
(6) reporting on the conclusions and the reasoning behind them.




Choices, assumptions, and approximations are present throughout the cycle.

**Mathematical Practices**

| | |
|---|---|
| 1: Make sense of problems and persevere in solving them. | High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, ―Does this make sense?‖ They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. |
| 2: Reason abstractly and quantitatively. | High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects. |
| 3: Construct viable arguments and critique the reasoning of others. | High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments |
| 4: Model with mathematics. | High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. |

| | |
|---|---|
| 5: Use appropriate tools strategically. | High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. |
| 6: Attend to precision. | High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions. |
| 7: Look for and make use of structure. | By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures. |
| 8: Look for and express regularity in repeated reasoning. | High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. |