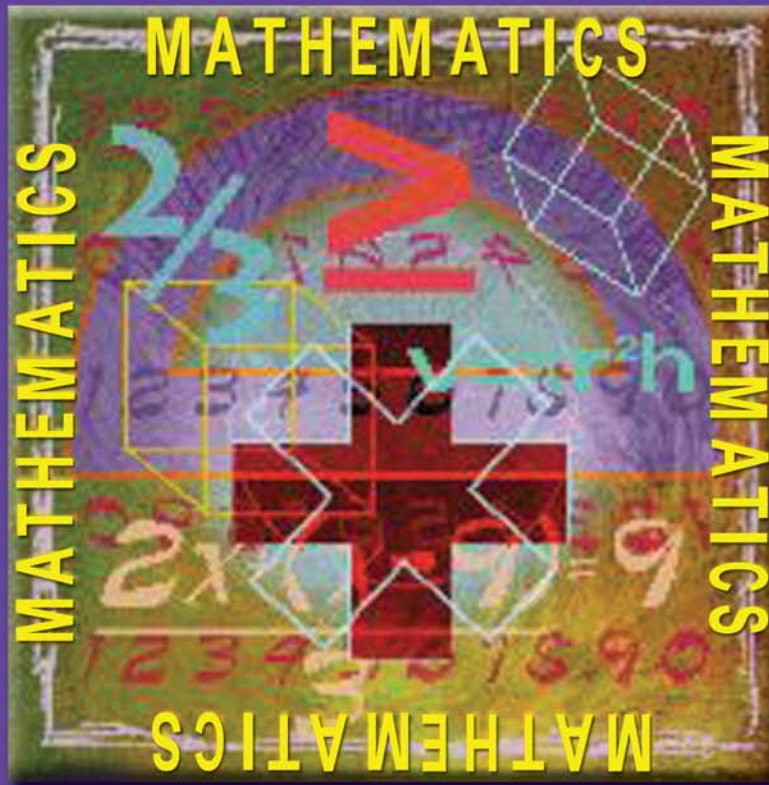


Proposed

AERO

Mathematics:

A Crosswalk of AERO
and the Common Core



Mathematics: Crosswalk AERO and the Common Core

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Introduction

AERO K-8 Mathematics Curriculum Framework

When curriculum standards were first developed in the 1990's, they were typically organized into grade spans. The goal was to allow curriculum flexibility with the idea that a student was expected to understand the concept, "By the End of Grade..." The general guide for the grade placement of standards within grade bands was perceived beliefs within the content community about when and how the big ideas of a discipline unfolded. State and national standards were in turn used by educators to develop aligned curriculum frameworks and later, accountability systems.

In light of current information being derived from cognitive research, the sequential arrangement of topics and concepts that are embedded in broad standards is being rethought. These ideas are now being mapped into learning progressions,¹ webs of interconnected ideas that illustrate how over time, student's ideas deepen and become more sophisticated and do so in predictable ways. Research-based learning progressions enable us to reframe how we think about Standards and Learning Expectations for students and to view them from a long-range perspective.

The AERO Mathematics Framework grade level indicators (K-8) reflect the current research on learning progressions. These grade level indicators are performance descriptors that indicate what typical progress towards the standards may look like. Using the metaphor of a ladder helps to illustrate the importance of the mastery of precursor ideas and skills as essential for making the cognitive leaps that results in a deep and flexible understanding of the standards. These progression points inform discussions about curriculum planning and can assist educators in making judgments about student progress towards a standard.

The AERO Mathematics Framework does not tell teachers how to teach rather helps guide teachers on the knowledge and skills students should have so they can create the best lessons and learning environments. These grade level indicators also help students and parents by setting clear and realistic goals for success. The Framework allows schools to move the conversation from the mathematics content to engaging discussions about instructional effectiveness.

¹ A representative set of incremental and interconnected stages in an individual's development of understanding about a specific concept or topic that, over time, evolves toward increasingly complex ways of thinking.

FAQ Common Core Standards

What is the Common Core State Standards Initiative?

The Common Core State Standards Initiative is a voluntary effort through the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA) Center for Best Practices to develop a common core of standards that are aligned with college and work expectations, include rigorous content and skills, and are internationally benchmarked. The intent is that these standards will be aligned to state assessment and classroom practice. Forty-eight states signed a memorandum of understanding to participate in the Common Core State Standards Initiative.

What content areas and grade levels are included in the Common Core Standards?

There are two sets of Common Core standards, spanning Kindergarten through grade 12—English-language arts and math. English-language arts and math were the first subjects chosen because these two subject areas provide core skills upon which students build skill sets in other subject areas. They are also the subjects most frequently assessed for accountability purposes.

Why do we need Common Core Standards?

Standards ensure that all students, no matter where they live, have the opportunity to be prepared for success in postsecondary education and the workforce. Currently, education standards are developed and implemented on a state-by-state basis.

Common standards will help ensure that all students are receiving a high-quality education consistently, from school-to-school and state-to-state. Common standards will provide a greater opportunity to share experiences and best practices within and across states that will improve the ability to best serve the needs of students.

What criteria was used to develop the Common Core Standards?

The standards:

- Are aligned with college and work expectations;
- Are clear, understandable and consistent;
- Include rigorous content and application of knowledge through high-order skills;
- Build upon strengths and lessons of current state standards;
- Are informed by other top performing countries, so that all students are prepared to succeed in a global economy and society; and
- Are evidence-based.

Who was involved in developing the Common Core Standards?

To create high-quality education standards, development occurred in three phases by groups that included teachers, parents, administrators, and educational experts, as well as select governors and state school officers. Teachers have been a critical voice in the development of the standards. The National Education Association (NEA), American Federation of Teachers (AFT), National Council of Teachers of Mathematics (NCTM), and National Council of Teachers of English (NCTE), among other organizations have been instrumental in bringing together teachers to provide specific, constructive feedback on the standards.

How does the K-8 AERO Framework align with the Common Core Standards?

The AERO Curriculum Framework is written in language more applicable to assessment. The Common Core Standards are written from an instructional standpoint.(See chart below) Both articulate what students should know/understand and be able to do to be prepared for entry-level college courses and career training programs.

Common Core	AERO
<p>5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.)</p> <p>$3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.)</p> <p>Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</p>	<p>Use and explain the operations of multiplication and division including the properties (e.g., identity element of multiplication, commutative property, property of zero, associative property, inverse operations).</p>

Similar to AERO's current standards and grade level expectations, the Common Core Standards do not represent a curriculum. Instead they are learning goals that students reach only when supported by a rich and engaging curriculum that infuses the mathematical practices outlined in the Standards document.

AERO has produced this crosswalk resource that presents the alignment of Common Core Standards and the AERO Mathematics Curriculum Framework. The intent of this crosswalk is to enhance understanding of how the Common Core State Standards in mathematics compares with current expectations AERO has for students and to inform thinking about the implementation of the Common Core Standards.

The K-8 Common Core Standards alignment documents are organized by grade level and "critical area" or AERO Grade level expectations. The "critical areas" as described in the common Core introductory sections for each grade level indicate the focus of the clusters of standards for the grade level and align closely with the AERO Standards and benchmarks.

The alignment shows a very good fit between our AERO Grade Level Expectations and Common Core Standards. There are areas, where the Common Core Standards clarify the intent of AERO Grade Level Expectations. while this suggests a misalignment, the intended rigor is not compromised.

The documents complement each other and provide focused and specific progressions of understandings, content knowledge, and skills that lead to college- and career-readiness. They define what students should “understand” as well as what students should be able to do to demonstrate understanding.

How does the AERO High School Framework align with the Common Core Standards?

The Common Core Standards for high school mathematics are organized under six conceptual categories, the equivalent of AERO’s strands in the discrete mathematics courses. Common Core Standards are further grouped in clusters that reflect topics in the AERO High School document. Of the 136 High School Mathematics Common Core Standards, 92 represent college- and career-readiness (CCR) and should be met by all students. The other 44 Common Core Standards have been designated STEM (Science, Technology, Engineering, Math) for students who wish to take advanced mathematics courses, such as calculus, advanced statistics or discrete mathematics (see table below). Some Common Core Standards designated as STEM are addressed in traditional Algebra II courses and a few reflect foundational knowledge addressed in Algebra I and Geometry.

Common Core Standards Category	College/Career Ready Required of ALL students	STEM Ready
Number and Quantity	9	18
Algebra	24	4
Functions	21	7
Geometry	37	6
Statistics and Probability	22	9
Modeling	Infused throughout all categories	
Total	92	44

Using both documents can benefit educators in informing instruction and planning assessments as they develop and implement a rigorous curriculum that promotes student learning and growth in student achievement.

CROSSWALK COMMON CORE AERO

Kindergarten

In Kindergarten, instructional time should focus on two critical areas:

- (1) representing, relating, and operating on whole numbers, initially with sets of objects;**
- (2) describing shapes and space.**

More learning time in Kindergarten should be devoted to number than to other topics (Common Core)

Common Core

AERO

Counting and Cardinality

Know number names and the count sequence.

1. Count to 100 by ones and by tens.

Count forward by 1's to 31 and backward from 20 with and without objects

CHANGE

Count forward by 1's and tens to 100 and backward from 100 with and without objects

2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Use a number line or chart to locate and identify the numbers (from 1 to 31) coming before/after a given number and between 2 given numbers

Change

Use a number line or chart to locate and identify the numbers (from 1 to 100) coming before/after a given number and between 2 given numbers

<p>3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). Count to tell the number of objects</p>	<p>Identify and read aloud numbers from 0 to at least 31</p> <p>Change Identify, write, and read aloud numbers from 0 to at least 31</p> <p>Count how many objects are in a set of up to 10 objects and count out a specific number of objects (up to 10) from a larger set.</p> <p>Change Count how many objects are in a set of up to 20 objects and count out a specific number of objects (up to 20) from a larger set.</p>
<p>4. Understand the relationship between numbers and quantities; connect counting to cardinality.</p> <p>a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.</p> <p>b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.</p> <p>c. Understand that each successive number name refers to a quantity that is one larger.</p>	<p>Use the ordinal numerals 1st through 10th to discuss positions in ordered lists</p> <p>Also see above</p>
<p>5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. Compare numbers.</p>	<p>Count how many objects are in a set of up to 10 objects and count out a specific number of objects (up to 10) from a larger set.</p> <p>CHANGE Count how many objects are in a set of up to 20 objects and count out a specific number of objects (up to 20) from a larger set.</p>

<p>6. Identify whether the number of objects in one group is greater than, less than, or</p>	<p>Compare two sets of up to 10 objects each and explain why the number of objects in</p>
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equal to the number of objects in another group, e.g., by using matching and counting strategies. ¹	one set is equal to, greater than, or less than the number of objects in the other set. CHANGE
7. Compare two numbers between 1 and 10 presented as written numerals.	Compare two sets of up to 20 objects each and explain why the number of objects in one set is equal to, greater than, or less than the number of objects in the other set.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	
1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Use concrete objects to model simple joining and separating situations (addition and subtraction) of whole numbers related to sums of 10 or less and write corresponding number sentence
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Model meanings of operations and the relationship between addition and subtraction (e.g., identity element of addition, commutative property) using manipulatives
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).	
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.	Use concrete objects to model simple joining and separating situations (addition and subtraction) of whole numbers related to sums of 10 or less and write corresponding number sentence Use drawings and labels to record solutions of addition and subtraction problems with answers less than or equal to 10
5. Fluently add and subtract within 5.	Not specifically addressed as being fluent

Number and Operations in Base Ten

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$);	Identify place value of each digit utilizing standard and expanded form through 20.
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understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	
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Measurement and Data	
Describe and compare measurable attributes.	
<p>1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.</p>	<p>Identify measurable attributes, such as length, weight, and capacity, and use these attributes to make direct comparisons</p> <p>Change Identify and describe measurable attributes, such as length, weight, and capacity, and use these attributes to make direct comparisons</p>
<p>2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.</p>	<p>Use comparative vocabulary in measurement settings (e.g., long/longer, short/shorter, more/less, hotter/colder, heavier/lighter, bigger/smaller)</p> <p>Determine and describe comparisons of length (longer, shorter, the same), mass (heavier, lighter, the same), and capacity (holds more, less, or about the same) using different-shaped or congruent containers, objects or figures.</p>
Classify objects and count the number of objects in each category.	
<p>3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (a Limit category counts to be less than or equal to 10.)</p>	<p>Sort objects into groups in one or more ways and identify which attribute was used to sort (size, shape, and color).</p> <p>Count how many objects are in a set of up to 20 objects and count out a specific number of objects (up to 20) from a larger set.</p> <p>Identify and create, compare and describe sets of objects as more, less or equal</p>

Geometry	
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).	
1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.	<p>Use relative position words including before/after, far/near, and over/under to place objects</p> <p>Describe the location of one object relative to another object using words such as in, out, over, under, above, below, between, next to, behind, and in front of</p> <p>Use the positional words near, far, below, above, beside, next to, across from, and between to describe the location of an object</p>
2. Correctly name shapes regardless of their orientations or overall size.	Identify two-dimensional shapes , i.e., circle, triangle, rectangles, and squares, regardless of size or orientation
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three- dimensional (“solid”). Analyze, compare, create, and compose shapes.	Identify three-dimensional figures in the environment.
4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).	<p>Sort objects into groups in one or more ways and identify which attribute was used to sort (size, shape, and color).</p> <p>Note: :teachers should include geometric figures in the learning opportunities</p>

<p>5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.</p>	<p>Not specifically identified as a progression, rather part of the learning opportunities which lead to identifying and describing two and three dimensional figures</p>
<p>6. Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”</p>	

CROSSWALK COMMON CORE AERO

GRADE 1

In Grade 1, instructional time should focus on four critical areas:

- (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20;**
- (2) developing understanding of whole number relationships and place value, including grouping in tens and ones;**
- (3) developing understanding of linear measurement and measuring lengths as iterating length units; and**
- (4) reasoning about attributes of, and composing and decomposing geometric shapes.**

Common Core	AERO
Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	
1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	Use concrete objects to model the addition of two or three addends and subtraction of whole numbers related to sums less than 20 and write the corresponding number sentence

<p>2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.</p>	<p>Solve and create a story problem that matches an addition or subtraction expression or equation using physical objects, pictures, or words.</p>
<p>Understand and apply properties of operations and the relationship between addition and subtraction.</p>	
<p>3. Apply properties of operations as strategies to add and subtract.3 Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)</p>	<p>Use the concept of commutative [$4 + 2 = 2 + 4$], associative [$(4 + 3) + 7 = 4 + (3 + 7)$], and identity [$0 + 3 = 3$] properties of addition to solve problems involving basic facts.</p>
<p>4. Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8</p>	

<p style="text-align: center;">Add and subtract within 20.</p>	
<p>5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).</p>	<p>Count forward by 1's to 120, with and without objects and starting with any number less than 120, and count by two's to at least 100</p>
<p>6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).</p>	<p>Recall from memory single digit addition facts (to $9 + 9$) and the corresponding subtraction facts</p> <p>Apply strategies, including counting on, counting back, and doubling, for addition facts to at least 10.</p> <p>Use movement on the number line to demonstrate the inverse relationship between addition and subtraction</p>

Work with addition and subtraction equations.	
7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$, $7 = 8 - 1$, $5 + 2 = 2 + 5$, $4 + 1 = 5 + 2$.	Model situations and solve equations that require addition and subtraction of whole numbers; use objects, pictures, and symbols
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 + ? = 11$, $5 = \diamond - 3$, $6 + 6 = \diamond$.	Select and/or write number sentences to find the unknown in problem-solving contexts involving single-digit addition and subtraction using appropriate labels

Number and Operations in Base Ten	
Extend the counting sequence.	
1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	Count forward by 1's to 99 and backward from 20 with and without objects and count by two's to at least 100 Change Count forward by 1's to 120, starting at any number less than 120 with and without objects and count by two's to at least 100
Understand place value.	
2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones)	Construct models and identify place value of each digit utilizing standard and expanded form through 99
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$. Use place value understanding and properties of operations to add and subtract.	Write, compare, and order numbers to at least 100 using the words equal to, greater than, less than, greatest, and least when appropriate. Change

	<p>Write, compare, and order numbers to at least 100 using the words equal to, greater than, less than, greatest, and least when appropriate.</p> <p>recording the results of comparisons with the symbols $>$, $=$, and $<$. When appropriate</p>
<p>4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.</p>	<p>Use concrete objects to model the addition of two or three addends and subtraction of whole numbers related to sums less than 20 and write the corresponding number sentence</p> <p>Change</p> <p>Use concrete objects to model the addition of two or three addends and subtraction of whole numbers related to sums less than 20 and write the corresponding number sentence.</p> <p>Using concrete models or drawings and strategies based on place value, add within 100, including adding a two-digit number and a one-digit number.</p>
<p>5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</p>	<p>Given a number and number line/hundreds chart, identify the nearest ten</p>
<p>6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>	

Measurement and Data	
Measure lengths indirectly and by iterating length units.	
1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.	Estimate and verify by measuring, length, weight, and capacity using nonstandard units (e.g., sticks, paper clips, blocks, beans) Identify the appropriate tool used to measure length (i.e., ruler), weight (i.e., scale), time (i.e., clock, calendar) and temperature (i.e., thermometer)
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.	Compare, order, describe, and represent objects by length and weight Use a variety of non-standard units to measure length
Tell and write time.	
3. Tell and write time in hours and half-hours using analog and digital clocks.	Tell time to the hour and half-hour using digital and analog clocks

Represent and interpret data.	
4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another	Gather data and represent data using tallies, tables, picture graphs, and bar-type graphs Analyze and interpret data by using mathematical language such as more than, less than, etc.

Geometry	
Reason with shapes and their attributes.	
1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.	Name, sort, and sketch two-dimensional shapes (circles, triangles, rectangles including squares) regardless of orientation..
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.	Identify, name, and describe three-dimensional objects i.e. cubes and spheres, regardless of size or orientation Compose and decompose common two-dimensional figures.
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	Identify and name halves, thirds, and fourths as part of a whole and as part of a group using models

CROSSWALK COMMON CORE AERO

GRADE 2

In Grade 2, instructional time should focus on four critical areas:

- (1) extending understanding of base-ten notation;**
- (2) building fluency with addition and subtraction;**
- (3) using standard units of measure; and**
- (4) describing and analyzing shapes.**

Common Core

AERO

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Add and subtract within 20.

Generate and solve one step addition and subtraction problems based on practical situations

Change
Generate and solve one and two step addition and subtraction word problems based on practical situations

2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. Work with equal groups of objects to gain foundations for multiplication.

Carry out addition and subtraction mentally involving: 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Demonstrate the relationships between odd and even numbers in addition and subtraction such as, odd + odd = even or odd – even = odd

<p>4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.</p>	<p>Model, represent, and explain multiplication (products to 81) as a rectangular array , as repeated addition and skip counting, or as equal-sized moves on the number line and division as repeated subtraction, sharing and grouping</p>
<p>Number and Operations in Base Ten</p>	
<p>Understand place value.</p>	
<p>1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:</p> <p>a. 100 can be thought of as a bundle of ten tens — called a “hundred.”</p> <p>b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p>	<p>Construct models and identify place value of each digit utilizing standard and expanded form through 999.</p>
<p>2. Count within 1000; skip-count by 5s, 10s, and 100s.</p>	<p>Count by tens or hundreds forward and backward starting at any number from 1 to 999 and count by fives and tens to at least 100.</p> <p>Change Count by fives, tens and hundreds starting at any number from 1 to 999</p>
<p>3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p>	<p>Identify, read aloud and write numbers to 1000.</p> <p>Construct models and identify place value of each digit utilizing standard and expanded form through 999</p>

<p>4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>Compare and order numbers from 0 to at least 1,000 using the words equal to, greater than, less than, greatest, or least when appropriate.</p> <p>Change Compare and order numbers from 0 to at least 1,000 using the words equal to, greater than, less than, greatest, or least when appropriate. using $>$, $=$, and $<$ symbols to record the results of comparisons</p>
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<p align="center">Use place value understanding and properties of operations to add and subtract.</p>	
<p>5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>	<p>Demonstrate efficient procedures for adding and subtracting 2 and 3 digit whole numbers and explain why the procedures work on the basis of place value and number properties</p>
<p>6. Add up to four two-digit numbers using strategies based on place value and properties of operations.</p>	
<p>7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p>	
<p>8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p>	
<p>9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanation may be in word or pictures)</p>	
<p>Carry out addition and subtraction mentally involving: 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds</p>	<p>Model and justify the relationship between addition and subtraction (e.g., identity element of addition, associative property, commutative property, inverse operations, fact families).</p>

Measurement and Data	
Measure and estimate lengths in standard units.	
1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	Select the appropriate units for measuring time, length, weight, and temperature
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	
3. Estimate lengths using units of inches, feet, centimeters, and meters.	Select and use appropriate tools and units to measure length, time, capacity, and weight (e.g., scales for kilograms; rulers for centimeters; measuring containers for cup, and liters) thermometer in degrees Celsius
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit	
Relate addition and subtraction to length.	
5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	<p>Carry out addition and subtraction mentally involving: 3-digit numbers and ones; 3-digit numbers and tens; 3-digit numbers and hundreds</p> <p>Model situations and solve equations that involve the addition and subtraction of whole numbers.</p> <p>Select and use appropriate tools and units to measure length, time, capacity, and weight (e.g., scales for kilograms; rulers for centimeters; measuring containers for cup, and liters) thermometer in degrees Celsius</p>

<p>6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.</p>	<p>Use words, number lines, and models to compare, and order whole numbers through 999</p>
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Work with time and money.

<p>7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.</p>	<p>Tell time to the nearest quarter hour and 5 minute interval using digital and analog clocks</p> <p>Use elapsed time in one hour increments, beginning on the hour, to determine start, end, and elapsed time</p>
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<p>8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?</p>	<p>Determine the value of a given set of coins</p> <p>Use decimals to show money amounts</p>
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Represent and interpret data.

<p>9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</p>	<p>Identify real life situations to gather data over time;</p>
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<p>10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.</p>	<p>Use tables, pictographs, and bar graphs to represent data</p> <p>Interpret data presented in circle, line, and bar graphs and answer questions about the displayed situation.</p>
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Geometry

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.	Describe, sketch, and compare two-dimensional shapes (rhombus, square, triangle, trapezoid, rectangle, pentagon, hexagon, octagon, and decagon) regardless of orientation.. Identify, classify, and sort basic geometric figures by shape, size, and geometric attributes. E.g. cube, sphere, and cylinder, prism, pyramid, and cone.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	Predict the results of combining and subdividing polygons and circles. Use non-standard units to cover a given region
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	Identify that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.

CROSSWALK COMMON CORE AERO

GRADE 3

In Grade 3, instructional time should focus on four critical areas:

(1) developing understanding of multiplication and division and strategies for multiplication and division within 100;

(2) developing understanding of fractions, especially unit fractions (fractions with numerator 1);

(3) developing understanding of the structure of rectangular arrays and of area; and

(4) describing and analyzing two-dimensional shapes.

Common Core

AERO

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7 .

Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal “jumps” on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems

2. Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.

Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal “jumps” on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems

<p>3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹</p>	<p>Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal “jumps” on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems</p>
<p>4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each other equations $8 \times ? = 48$, $5 = \diamond \div 3$, $6 \times 6 = ?$.</p>	<p>Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit times one-digit multiplication using appropriate labels</p> <p>Model, explain, and solve open number sentences including addition, subtraction, and multiplication facts.</p>
<p>Understand properties of multiplication and the relationship between multiplication and division.</p>	
<p>5. Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.)</p> <p>$3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.)</p> <p>Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)</p>	<p>Use and explain the operations of multiplication and division including the properties (e.g., identity element of multiplication, commutative property, property of zero, associative property, inverse operations).</p>
<p>6. Understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.</p>	<p>Apply the inverse relationship between multiplication and division (e.g., $5 \times 6 = 30$, $30 \div 6 = 5$) and the relationship between multiples and factors.</p>
<p style="text-align: center;">Multiply and divide within 100.</p>	
<p>7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</p>	<p>Identify whole number factors and/or pairs of factors for a given whole number through 24.</p>

Solve problems involving the four operations, and identify and explain patterns in arithmetic.	
8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.³	Generate and solve two step addition and subtraction problems and one step multiplication problems based on practical situations
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	Recognize and describe patterns using objects and numbers found in tables, number charts, and charts
Number and Operations in Base Ten	
Use place value understanding and properties of operations to perform multi-digit arithmetic.	
1. Use place value understanding to round whole numbers to the nearest 10 or 100.	Round whole numbers through 10,000 to the nearest ten, hundred, and thousand and round fractions to the nearest whole number
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	Describe and show relationships between strategies and procedures for multiplying and dividing that involve addition and subtraction and explain strategies
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations	Name the number that is 10 more than or 100 more than any number from 0 through 9,999 and 10 less than or 100 less than any number from 100 through 10,000.

Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8)

Use concrete models and pictorial representations to demonstrate the meaning of fractions (proper and improper) as parts of a whole, parts of a set, and division by whole numbers through twelfths

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

<p>3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.</p> <p>a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</p> <p>b. Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</p> <p>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.</p> <p>d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>	<p>Identify, name and use equivalent fractions with denominators 2, 4 and 8</p>
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Measurement and Data	
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	
<p>1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.</p>	<p>Tell time to the nearest minute using digital and analog clocks</p> <p>Use elapsed time in half-hour increments, beginning on the hour or half-hour, to determine start, end, and elapsed time</p>
<p>2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (I).Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.</p>	<p>Compare, order, and describe objects by various measurable attributes for area and volume/capacity</p> <p>Select and use appropriate units of measure and measure to a required degree of accuracy (to the nearest $\frac{1}{2}$ unit)</p>
Represent and interpret data.	
<p>3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</p>	<p>Use graphical representations including number lines, frequency tables, and pictographs to represent data</p> <p>Compare data and interpret quantities represented on tables and different types of graphs (line plots, pictographs, and bar graphs), make predictions, and solve problems based on the information.</p>
<p>4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.</p>	<p>Estimate and measure length using fractional parts to the nearest $\frac{1}{2}$ unit in the Metric system</p>

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	
<p>5. Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.</p> <p>b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.</p>	<p>Estimate and measure perimeter and area, using links, tiles, grid paper, geoboards, and dot paper</p>
Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	
<p>6. Relate area to the operations of multiplication and addition.</p> <p>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.</p> <p>b. Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.</p> <p>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.</p> <p>d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</p>	<p>Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal “jumps” on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems</p>

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	
8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters	Estimate and measure perimeter and area, using links, tiles, grid paper, geoboards, and dot paper
Reason with shapes and their attributes.	
1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	Identify, describe, and classify: cube, sphere, prism, pyramid, cone, and cylinder in terms of the number and shape of faces, edges, and vertices. Describe, compare, analyze, and classify two-dimensional shapes by sides and angles
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape	Analyze the results of combining and subdividing circles, triangles, quadrilaterals, pentagons, hexagons, and octagons Use concrete models and pictorial representations to demonstrate the meaning of fractions (proper and improper) as parts of a whole, parts of a set, and division by whole numbers through twelfths

CROSSWALK COMMON CORE AERO

GRADE 4

In Grade 4, instructional time should focus on three critical areas:

(1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends;

(2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers;

(3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Common Core	AERO
Operations and Algebraic Thinking	
Use the four operations with whole numbers to solve problems.	
<p>1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p>	<p>Generate and solve addition, subtraction, multiplication, and division problems using whole numbers in practical situations</p>
<p>2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.1</p>	<p>Select and test algorithms used in computational situations that involve multiplication and division of whole numbers and explain strategies.</p> <p>Select and/or write number sentences (equations) to find the unknown in problem-solving contexts involving two-digit by one-digit division using appropriate labels</p>
<p>3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p>	
Gain familiarity with factors and multiples.	
<p>4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p>	<p>Identify factors of composite numbers less than 50</p> <p>Change Identify factors of composite numbers less than 100</p> <p>Use divisibility concepts to classify numbers as prime or composite</p>

Generate and analyze patterns.	
<p>5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</p>	<p>Create, describe, and extend growing and repeating patterns with physical materials and symbols including numbers.</p> <p>Identify and describe patterns resulting from operations involving even and odd numbers (such as even + even = even)</p>
Number and Operations in Base Ten	
Generalize place value understanding for multi-digit whole numbers.	
<p>1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</p>	<p>Model and identify place value of each digit utilizing standard and expanded form through 9999.</p>
<p>2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>Read and write numbers to at least 100,000</p> <p>Use symbols (i.e., $<$, $=$, $>$) and models to compare and order whole numbers through 9,999</p>
<p>3. Use place value understanding to round multi-digit whole numbers to any place.</p>	<p>Round whole numbers to 1,000,000 to any place value and round decimals to the nearest whole, 10^{th}, or 100^{th} place.</p>

Use place value understanding and properties of operations to perform multi-digit arithmetic.	
4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.	Add and subtract whole numbers (up to five -digit number)
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	<p>Represent multiplication of two-digit by two digit numbers and describe how that representation connects to the related number sentence.</p> <p>Change Represent multiplication of up to four-digit by one digit numbers and describe how that representation connects to the related number sentence.</p> <p>Apply models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive).</p> <p>Change Apply models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive).</p> <p>Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors</p>

<p>6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>	<p>Apply models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive).</p> <p>Change Apply models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive).</p> <p>Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors</p>
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<p align="center">Number and Operations</p>	
<p align="center">Extend understanding of fraction equivalence and ordering.</p>	
<p>1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>	<p>Compare and order positive fractions (including positive mixed numbers) and decimals on the number line, in number</p>
<p>2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.</p>	

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

**b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples:
 $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.**

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem (addition and subtraction with unlike denominators in general is not a requirement)

Compare and order positive fractions (including positive mixed numbers) and decimals on the number line, in number sentences, and in lists

In Grade 3

Use concrete models to add and subtract simple common fractions with the same denominator.

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 For example, express $\frac{3}{10}$ as $\frac{30}{100}$, and add $\frac{3}{10} + \frac{4}{100} = \frac{34}{100}$.	Write a fraction equivalent to a given fraction using common multiples. And simplify fractions using common factors.
6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	Use models to connect and compare equivalent fractions and decimals.
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.	Compare and order positive fractions(including positive mixed numbers) and decimals on the number line, in number sentences, and in lists

Measurement and Data	
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.	
<p>1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</p>	<p>Estimate and convert units of measure for length, area, and weight with the same measurement system (metric)</p>
<p>2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p>	<p>Use appropriate tools to determine, estimate, and compare units for measurement of weight/m)ass, area, size of angle (using the benchmark angles 45°, 90°, 180°, 270°, and 360, temperature, length, distance, and volume in metric systems and time in real-life situations.</p>
<p>3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</p>	<p>Measure length, area, weight , and temperature, to a required degree of accuracy in metric systems</p> <p>Describe relationships of rectangular area to numerical multiplication</p>
Represent and interpret data.	
<p>4. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</p>	<p>Compare data and interpret quantities represented on tables and graphs including line graphs, bar graphs, frequency tables, and stem-and-leaf plots to make predictions and solve</p>

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Identify, draw, and describe horizontal, vertical, and oblique lines

Identify, draw, and classify angles, including straight, right, obtuse, and acute.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

Identify, draw, and describe horizontal, vertical, and oblique lines

Identify, draw, and classify angles, including straight, right, obtuse, and acute.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Analyze and describe the similarities and differences between and among two dimensional geometric shapes, figures, and models using mathematical language

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Identify shapes that are congruent, similar, and/or symmetrical using a variety of methods including transformational motions

CROSSWALK COMMON CORE AERO**GRADE 5**

In Grade 5, instructional time should focus on three critical areas:

(1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions);

(2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and

(3) developing understanding of volume.

Common Core**AERO****Operations and Algebraic Thinking****Write and interpret numerical expressions.**

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Devise a rule for an input/output function table, describing it in words and symbols

2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product. Analyze patterns and relationships.

<p>3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</p>	<p>Identify a rule for a pattern involving addition, subtraction, or multiplication</p> <p>Interpret and write a rule for a one operation function table Ex. Adding 3</p>
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Number and Operations in Base Ten

Understand the place value system.

<p>1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</p>	<p>Recall from memory multiplying and dividing by 10, 100, and 1,000</p>
<p>2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p>	<p>Recall from memory multiplying and dividing by 10, 100, and 1,000</p>
<p>3. Read, write, and compare decimals to thousandths.</p> <p>a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.</p> <p>b. Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.</p>	<p>Read, write, compare, and order all whole numbers, fractions, mixed numbers and decimals using multiple strategies (e.g. symbols, manipulatives, number line, and place value concepts)</p>
<p>4. Use place value understanding to round decimals to any place.</p>	

Perform operations with multi-digit whole numbers and with decimals to hundredths.	
5. Fluently multiply multi-digit whole numbers using the standard algorithm.	<p>Add, subtract, multiply, and divide (with and without remainders) using non-negative rational numbers.</p> <p>Multiply four-digit numbers by two-digit numbers (including whole numbers and decimals).</p>
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	<p>Add, subtract, multiply, and divide (with and without remainders) using non-negative rational numbers.</p>
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	<p>Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators</p> <p>Multiply four-digit numbers by two-digit numbers (including whole numbers and decimals).</p> <p>Change Multiply and four-digit numbers by two-digit numbers (including whole numbers and decimals).</p> <p>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value</p>

Number and Operations	
Use equivalent fractions as a strategy to add and subtract fractions.	
<p>1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)</p>	<p>Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators</p>
<p>2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$</p>	<p>Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators</p>
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	
<p>3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</p>	<p>Identify and represent ratios as comparisons of part-to-part and part-to-whole relationships, and solve problems involving ratios.</p>

<p>4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</p> <p>a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)</p> <p>b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</p>	<p>Multiply four-digit numbers by two-digit numbers (including whole numbers and decimals).</p> <p>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value</p> <p>Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators</p> <p>Change Multiply four-digit numbers by two-digit numbers (including whole numbers, and decimals).</p> <p>Add, subtract, multiply, and divide, fractions and decimals to hundredths, using concrete models or drawings and strategies based on place value</p>
<p>5. Interpret multiplication as scaling (resizing), by:</p> <p>a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</p> <p>b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.</p>	
<p>6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</p>	

<p>7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)</p> <p>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.</p> <p>b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.</p> <p>c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$-cup servings are in 2 cups of raisins?</p>	<p>Multiply four-digit numbers by two-digit numbers (including whole numbers and decimals).</p> <p>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value</p> <p>Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators</p> <p>Change Multiply four-digit numbers by two-digit numbers (including whole numbers, and decimals).</p> <p>Add, subtract, multiply, and divide, fractions and decimals to hundredths, using concrete models or drawings and strategies based on place value</p>
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Measurement and Data	
Convert like measurement units within a given measurement system.	
<p>1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p>	<p>Estimate and convert units of measure for weight and volume/capacity within the same measurement system (metric)</p>
Represent and interpret data.	
<p>2. Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</p>	<p>Use a variety of graphical representations including including line graphs, stem-and-leaf plots, histograms, and box-and-whisker plots to organize and represent data</p>
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.	
<p>3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</p> <p>a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.</p> <p>b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.</p>	<p>Measure volume and weight to a required degree of accuracy in the metric systems</p>
<p>4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</p>	

<p>5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</p> <p>a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</p> <p>b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</p> <p>c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</p>	<p>Measure volume and weight to a required degree of accuracy in the metric systems</p>
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Geometry	
Graph points on the coordinate plane to solve real-world and mathematical problems.	
<p>1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).</p>	<p>Identify, draw, label, and describe planes, parallel lines, intersecting lines, and perpendicular lines</p>
<p>2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.</p>	<p>Graph coordinates representing geometric shapes in the first quadrant</p>
Classify two-dimensional figures into categories based on their properties.	
<p>3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</p>	<p>Analyze and describe the characteristics of symmetry relative to classes of polygons (parallelograms, triangles, etc.).</p>
<p>4. Classify two-dimensional figures in a hierarchy based on properties.</p>	

CROSSWALK COMMON CORE AERO

GRADE 6

In Grade 6, instructional time should focus on four critical areas:

(1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems;

(2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers;

(3) writing, interpreting, and using expressions and equations; and

(4) developing understanding of statistical thinking.

Common Core	AERO
Ratios and Proportional Relationships	
Understand ratio concepts and use ratio reasoning to solve problems.	
<p>1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</p>	<p>Compare quantities and solve problems using ratios, rates and percents.</p>
<p>2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”¹</p>	<p>Write and apply ratios in mathematical and practical situations involving measurement and monetary conversions</p>

<p>3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.</p> <p>a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.</p> <p>b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</p> <p>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.</p> <p>d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities</p>	<p>Represent percents in various forms using numbers, pictures, models, or circle graphs and solve problems involving percentages. Greater than 100 and less than 1.</p>
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The Number System	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	
<p>1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.) How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of and with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?</p>	<p>Apply the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and justify why they work</p>
Compute fluently with multi-digit numbers and find common factors and multiples.	
<p>2. Fluently divide multi-digit numbers using the standard algorithm.</p>	<p>Apply the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and justify why they work</p>
<p>3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p>	<p>Model addition and subtraction of integers with physical materials and the number line.</p> <p>Model addition and subtraction of mixed numbers with and without regrouping and fractions with like and unlike denominators</p>

<p>4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4(9 + 2)$.</p>	<p>Identify the greatest common factor for a set of whole numbers.</p>
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Apply and extend previous understandings of numbers to the system of rational numbers.

<p>5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p>	<p>Compare positive fractions, decimals, and positive and negative integers using symbols (i.e., $<$, $=$, $>$) and number lines</p>
<p>6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.</p> <p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p>	<p>Read, write, compare, and order integers using multiple strategies (e.g., symbols, manipulatives, number line).</p>

<p>7. Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.</p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3\text{ }^{\circ}\text{C} > -7\text{ }^{\circ}\text{C}$ to express the fact that $-3\text{ }^{\circ}\text{C}$ is warmer than $-7\text{ }^{\circ}\text{C}$.</p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.</p> <p>d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.</p>	<p>Explain the meaning and relationship between absolute value and opposites.</p>
<p>8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>	<p>Draw and label the components of the coordinate plane; i.e., coordinates, quadrants, origin, x- and y-axes</p>

Expressions and Equations	
Apply and extend previous understandings of arithmetic to algebraic expressions.	
<p>1. Write and evaluate numerical expressions involving whole-number exponents.</p>	<p>Evaluate formulas and algebraic expressions using whole number values</p> <p>Formulate algebraic expressions, equations, and inequalities to reflect a given situation.</p>
<p>2. Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.</p> <p>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</p> <p>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.</p>	<p>Determine the rule, output or input; given an input/output model using one operation, write an algebraic expression for the rule and use to identify other input/output values.</p> <p>Represent the relationship in an input-output situation using a simple equation, graph, table, or word description</p> <p>Write simple expressions and equations using variables to represent mathematical situations</p>

<p>3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.</p>	<p>Determine the rule, output or input; given an input/output model using one operation, write an algebraic expression for the rule and use to identify other input/output values.</p> <p>Represent the relationship in an input-output situation using a simple equation, graph, table, or word description</p>
<p>4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.</p>	
<p>Reason about and solve one-variable equations and inequalities.</p>	
<p>5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p>	<p>Solve simple equations using guess-and-check, diagrams, properties, or inspection, explaining the process used.</p> <p>Change Solve simple equations and inequalities using guess-and-check, diagrams, properties, or inspection, explaining the process used.</p>
<p>6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p>	
<p>7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.</p>	
<p>8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p>	<p>Formulate algebraic expressions, equations, and inequalities to reflect a given situation.</p>

Represent and analyze quantitative relationships between dependent and independent

variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Write simple expressions and equations using variables to represent mathematical situations

When given a rule relating two variables create a table and represent the ordered pairs on a coordinate plane

Geometry	
Solve real-world and mathematical problems involving area, surface area, and volume.	
<p>1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>Describe the characteristics of right, acute, obtuse, scalene, equilateral, and isosceles triangles</p> <p>Change Describe the characteristics of right, acute, obtuse, scalene, equilateral, and isosceles triangles and determine the area of a right triangle</p> <p>Identify, classify, compare, and draw regular and irregular quadrilaterals</p> <p>Construct three-dimensional figures using manipulatives and generalize the relationships among vertices, faces, and edges (such as Euler's Formula).</p>
<p>2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l w h$ and $V = b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p>	<p>Make a two-dimensional drawing of a three-dimensional figure</p> <p>Change Make a two-dimensional drawing of a three-dimensional figure to visualize and solve problems; e.g., those involving surface area and volume</p>
<p>3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>Draw and label the components of the coordinate plane; i.e., coordinates, quadrants, origin, x- and y-axes</p>

<p>4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>Make a two-dimensional drawing of a three-dimensional figure</p> <p>Change Make a two-dimensional drawing of a three-dimensional figure to visualize and solve problems; e.g., those involving surface area and volume</p>
<p>Statistics and Probability</p>	
<p>Develop understanding of statistical variability.</p>	
<p>1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.</p>	<p>Formulate questions that guide the collection of data</p>
<p>2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</p>	<p>Analyze and solve application problems involving measures of central tendency (mean, median, mode) and dispersion (range) from data, graphs, tables, and experiments use appropriate technology to compare two sets of data</p>
<p>3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p>	<p>Interpret and explain line graphs, double bar graphs, frequency plots, stem-and-leaf plots, histograms, and box-and-whisker plots.</p>

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	Analyze various representations of a set of data to draw conclusions and make predictions.
5. Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	Interpret and explain line graphs, double bar graphs, frequency plots, stem-and-leaf plots, histograms, and box-and-whisker plots.

CROSSWALK COMMON CORE AERO

GRADE 7

In Grade 7, instructional time should focus on four critical areas:

- (1) developing understanding of and applying proportional relationships;**
- (2) developing understanding of operations with rational numbers and working with expressions and linear equations;**
- (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and**
- (4) drawing inferences about populations based on samples.**

Common Core	AERO
Ratios and Proportional Relationships	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	
1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour	Solve problems using ratios, rates and percents. Use unit rates (e.g., miles per hour, words per minutes) to solve problems

<p>2. Recognize and represent proportional relationships between quantities.</p> <p>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.</p> <p>b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</p> <p>c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.</p> <p>d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.</p>	<p>Write and apply proportions to solve mathematical and practical problems involving measurement and monetary conversions</p>
<p>3. Use proportional relationships to solve multistep ratio and percent. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</p>	

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

b. Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

Solve equations that represent algebraic and real-world problems using multiple methods including the real number properties.

<p>2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p>a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.</p> <p>b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.</p> <p>c. Apply properties of operations as strategies to multiply and divide rational numbers.</p> <p>d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p>	<p>Use the concepts of number theory, including prime and composite numbers, factors, multiples, and the rules of divisibility to solve problems</p>
<p>3. Solve real-world and mathematical problems involving the four operations with rational numbers.</p>	

Expressions and Equations

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Use the order of operations to simplify and/or evaluate whole numbers (including exponents and grouping symbols).

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”

Solve real-life and mathematical problems using numerical and algebraic expressions and equations

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.

Solve equations that represent algebraic and real-world problems using multiple methods including the real number properties.

<p>4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</p> <p>a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</p> <p>b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.</p>	<p>Solve and graphically represent equations and inequalities in one variable with integer solutions</p>
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Geometry	
Draw, construct, and describe geometrical figures and describe the relationships between them.	
1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	Make a model of a three-dimensional figure from a two-dimensional drawing.
2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	Classify triangles based on side and angle measurements; Change Classify and construct triangles based on side and angle measurements; Construct and identify congruent angles, parallel lines, and perpendicular lines
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Construct two-dimensional representations of three-dimensional objects. Build and sketch three-dimensional solids; e.g., using nets, manipulatives.
4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	Estimate and compare corresponding units of measure for area and volume/capacity metric systems Select and use appropriate tools and units to determine the measurements needed for calculating perimeter, circumference, area, surface area, and volume

<p>5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p>	<p>Find missing angle measurements for parallel lines cut by a transversal(s) and for a vertex of a polygon.</p>
<p>6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>	<p>Use formulas and strategies, such as decomposition, to compute the perimeter and area of triangles, parallelograms, trapezoids, the circumference and area of circles, and find the area of more complex shapes.</p>
<p>Statistics and Probability</p>	
<p>Use random sampling to draw inferences about a population.</p>	
<p>1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p>	<p>Identify a real life situation using statistical measures (mean, median, mode, range, outliers) overtime, make a hypothesis as to the outcome; design and implement a method to collect, organize and analyze data; analyze the results to make a conclusion; evaluate the validity of the hypothesis based upon collected data, design a mode of presentation using words, graphs, models, and/or tables</p>
<p>2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</p>	

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Identify a real life situation using statistical measures (mean, median, mode, range, outliers) overtime, make a hypothesis as to the outcome; design and implement a method to collect, organize and analyze data; analyze the results to make a conclusion; evaluate the validity of the hypothesis based upon collected data, design a mode of presentation using words, graphs, models, and/or tables

Investigate chance processes and develop, use, and evaluate probability models.

<p>5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</p>	<p>Determine theoretical probability of an event, make and test predictions through experimentation.</p>
<p>6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.</p>	<p>Determine and explain whether a real-world situation involves permutations or combinations, then use appropriate technology to solve the problem.</p>

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Determine theoretical probability of an event, make and test predictions through experimentation.

Determine and explain whether a real-world situation involves permutations or combinations, then use appropriate technology to solve the problem.

<p>8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p> <p>a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p> <p>b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.</p> <p>c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?</p>	<p>Determine theoretical probability of an event, make and test predictions through experimentation.</p> <p>Determine and explain whether a real-world situation involves permutations or combinations, then use appropriate technology to solve the problem.</p>
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CROSSWALK COMMON CORE AERO

GRADE 8

In Grade 8, instructional time should focus on three critical areas:

(1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations;

2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

Common Core

AERO

The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Analyze, describe and compare the characteristics of rational and irrational numbers.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Represent and compare rational and irrational numbers symbolically and on a number line.

Expressions and Equations	
Work with radicals and integer exponents.	
1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $32 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	Recognize and appropriately use exponential and scientific notation.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	Use rational and irrational numbers to solve real-world and mathematical problems.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.	
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	Represent numbers using scientific notation in mathematical and practical situations.
Understand the connections between proportional relationships, lines, and linear equations.	Solve contextual problems using ratios, rates, or percents and verify the reasonableness of the solution.

<p>5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</p>	<p>Solve contextual problems using ratios, rates, or percents and verify the reasonableness of the solution.</p> <p>Apply ratios and proportions to calculate rates and solve mathematical and practical problems using indirect measure.</p>
<p>6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</p>	

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Solve linear equations and inequalities and represent the solution graphically

<p>8. Analyze and solve pairs of simultaneous linear equations.</p> <p>a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</p> <p>b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</p> <p>c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</p>	<p>In AERO Algebra I statements</p>
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Functions	
Define, evaluate, and compare functions.	
<p>1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p>	<p>Identify, model, and describe linear functions</p> <p>Translate among verbal descriptions, graphic, tabular, and algebraic representations of mathematical situations</p>
<p>2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</p>	
<p>3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.</p>	<p>Solve linear equations and inequalities and represent the solution graphically</p>
<p>Use functions to model relationships between quantities.</p>	<p>Identify, model, and describe linear functions</p> <p>Translate among verbal descriptions, graphic, tabular, and algebraic representations of mathematical situations</p>
<p>4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p>	

<p>5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>	<p>Translate among verbal descriptions, graphic, tabular, and algebraic representations of mathematical situations</p>
<p>Geometry</p>	
<p>Understand congruence and similarity using physical models, transparencies, or geometry software.</p>	
<p>1. Verify experimentally the properties of rotations, reflections, and translations:</p> <p>a. Lines are taken to lines, and line segments to line segments of the same length.</p> <p>b. Angles are taken to angles of the same measure.</p> <p>c. Parallel lines are taken to parallel lines.</p>	<p>Draw the results of a combination of transformations in the coordinate plane; i.e., reflections, rotations, and translations</p>
<p>2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p>	<p>Draw the results of a combination of transformations in the coordinate plane; i.e., reflections, rotations, and translations</p>

<p>3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p>	<p>Draw the results of a combination of transformations in the coordinate plane; i.e., reflections, rotations, and translations</p>
<p>4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p>	<p>Use two-dimensional representations (nets) of three-dimensional objects to describe objects from various perspectives.</p> <p>Apply the properties of equality and proportionality To find missing attributes of congruent or similar shapes</p>
<p>5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.</p>	<p>Find and verify the sum of the measures of interior angles of triangles</p>
<p>Understand and apply the Pythagorean Theorem.</p>	<p>Explain the Pythagorean Theorem and apply it to solve routine and non-routine problems.</p>
<p>6. Explain a proof of the Pythagorean Theorem and its converse.</p>	
<p>7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p>	
<p>8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>	

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	
9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	Use formulas and/or appropriate measuring tools to find length and angle measures (to appropriate levels of precision), perimeter, area, volume, and surface area of polygons, circles, spheres, cones, pyramids, and composite or irregular figures.
Investigate patterns of association in bivariate data.	
1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	Use appropriate graphical representations to organize, display, and read data .
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	

<p>3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</p>	<p>Calculate slope, midpoint, and distance using equations and formulas</p>
<p>4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</p>	<p>Draw inferences, make conjectures and construct convincing arguments involving different effects that changes in data values have on measures of central tendency misuses of statistical or numeric information, based on data analysis of same and different sets of data</p> <p>Select and apply appropriate measures of data distribution using interquartile range and central tendency</p>

**Alignment
of
AERO High School Proposed Standards
to
The Common Core**

Key

A-1 refers to Algebra I standards
A-2 refers to Algebra 2 standards
Geom refers to Geometry standards
Precalculus refers to Precalculus standards

NOTE

There is a difference between the intent of the AERO documents and the Common Core. The AERO document was developed from the standpoint of assessment and uses performance indicators and reflects the math all students should know, understand, and be able to do. The Common Core was developed from the standpoint of instruction and not only includes the math required of all students but also describes additional mathematics that students should learn to pursue careers and majors in science, technology, engineering and mathematics (STEM) fields.

Standards beyond the college and career readiness level that are necessary for STEM careers are prefixed with a symbol STEM, as in this example

The AERO standards address most of the Common Core and the exceptions have been noted. The final AERO standards for the high school will include a clarifying document that will incorporate the language of the Common Core.

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	The Real number system	Extend the properties of exponents to rational exponents	1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5(1/3)^3$ to hold, so $(5^{1/3})^3$ must equal 5.	A-1 7.5 a Use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents
			2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
		Classify numbers as rational or irrational	3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	Quantities	Reason quantitatively and use units to solve problems	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	See AERO Process Standards
	2. Define appropriate quantities for the purpose of descriptive modeling.			
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.				
The Complex Number	Perform arithmetic operations with complex numbers	1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	A-2 7.3 a Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane.	
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.				
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.				

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	The Complex Number	Represent complex numbers and their operations on the complex plane	4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	<p>PreCalculus 4.1 a Define and use polar coordinates and relate polar coordinates to Cartesian coordinates .</p> <p>PreCalculus 4.1 b Represent equations given in Cartesian coordinates in terms of polar coordinates .</p> <p>PreCalculus 4.1 c Graph equations in the polar coordinate plane .</p> <p>PreCalculus 4.1 d Define complex numbers, convert complex numbers to polar form and multiply complex numbers in polar form</p>
			5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3} i)^3 = 8$ because $(-1 + \sqrt{3} i)$ has modulus 2 and argument 120° .	<p>A-2 7.3 a Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane.</p>
			6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	The Complex Number	Use complex numbers in polynomial identities and equations	7. Solve quadratic equations with real coefficients that have complex solutions.	A-2 7.3 b Solve quadratic equations in the complex number system.
			8. (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.	
			9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials	

Topic	Concept	Common Core		AERO
	Vector and Matrix Quantities	Represent and model with vector quantities.	1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v , $ v $, $\ v\ $, v).	STEM
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.				

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	Vector and Matrix Quantities	Perform operations on vectors.	<p>4. (+) Add and subtract vectors.</p> <p>Add vectors end-to-end, component-wise, and by the parallelogram rule.</p> <p>Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.</p> <p>Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.</p> <p>Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction.</p> <p>Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</p>	STEM

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	Vector and Matrix Quantities	Perform operations on vectors.	<p>5. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.</p> <p>Compute the magnitude of a scalar multiple cv using $\ cv\ = c v$. Compute the direction of cv knowing that when $c v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).</p>	STEM

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	Vector and Matrix Quantities	Perform operations on matrices and use matrices in applications.	6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	<p>A-1 7.4 a Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.</p> <p>A-2 7.2 a Solve systems of linear equations and inequalities in three variables by substitution and elimination.</p> <p>7.2b Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.</p> <p>Matrices are STEM</p>
			7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled	
			8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	
			9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	

Topic	Concept	Common Core		AERO
NUMBER AND QUANTITY	Vector and Matrix Quantities	Perform operations on matrices and use matrices in applications.	10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse	A-2 7.2 a Solve systems of linear equations and inequalities in three variables by substitution and elimination. 7.2b Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.
			11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	STEM
			12. (+) Work with 2×2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.	A-2 7.2 a Solve systems of linear equations and inequalities in three variables by substitution and elimination. 7.2b Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.

Topic	Concept	Common Core		AERO
ALGEBRA	Seeking Structure in Expressions	Interpret the structure of expressions.	<p>1. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</p>	<p>A-1 7.5 a Use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.</p>
			<p>2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</p>	

Topic	Concept	Common Core		AERO
ALGEBRA	Seeking Structure in Expressions	Write expressions in equivalent forms to solve problems.	<p>3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>a. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</p>	<p>A-1 7.6 b Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square and by using the quadratic formula.</p> <p>A-2 7.6 c Solve exponential and logarithmic equations.</p> <p>A-2 7.6 d Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. Use technology as appropriate.</p>
			<p>4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.</p>	<p>A-2 7.7 d Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series. Interpret the solutions and determine whether the solutions are reasonable using spreadsheets as appropriate.</p>

Topic	Concept	Common Core		AERO
ALGEBRA	Arithmetic with Polynomial and Rational Expressions	Perform arithmetic operations on polynomials	1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	A-2 7.4 e Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.
		Understand the relationship between zeros and factors of polynomials.	2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	A-2 7.4 f Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression.
			3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	A-2 7.4 a Factor polynomials completely and solve polynomial equations by factoring. A-2 7.4 b Solve problems that can be represented or modeled using polynomial equations, interpret the solutions and determine whether the solutions are reasonable.

Topic	Concept	Common Core		AERO
ALGEBRA	Arithmetic with Polynomial and Rational Expressions	Use polynomial identities to solve problems.	4. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	A-2 7.4 a Factor polynomials completely and solve polynomial equations by factoring
			5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1	A-2 7.4 d Use the binomial theorem to expand binomial expressions raised to positive integer powers.

Topic	Concept	Common Core		AERO
ALGEBRA	Arithmetic with Polynomial and Rational Expressions	Rewrite rational expressions.	6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	A-2 7.5 b Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.
			7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions	

Topic	Concept	Common Core		AERO
ALGEBRA	Creating Equations	Create equations that describe numbers or relationships.	1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	A-1 7.2d Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable.
			2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	A-1 7.4 a Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.
			3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	
			4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .	An Application of A-1 7.2d Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable.

Topic	Concept	Common Core		AERO
ALGEBRA	Reasoning With Equations and Inequalities	Understand solving equations as a process of reasoning and explain the reasoning.	1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	<p>A-2 7.5 e Solve equations that contain radical expressions and identify extraneous roots when they occur.</p> <p>A-2 7.5 f Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions and determine whether the solutions are reasonable</p>
			2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	

Topic	Concept	Common Core		AERO
ALGEBRA	Reasoning With Equation and Inequalities	Solve equations and inequalities in one variable.	3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	A-1 7.2d Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable.
			<p>4. Solve quadratic equations in one variable.</p> <p>Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</p> <p>Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</p>	<p>A-2 7.3 b Solve quadratic equations in the complex number system.</p> <p>A-2 7.3 c Solve problems that can be modeled using quadratic equations and functions, interpret the solutions, and determine whether the solutions are reasonable.</p>

Topic	Concept	Common Core		AERO
ALGEBRA	Reasoning With Equation and Inequalities	Solve systems of equations.	5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	A-1 7.4 a Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.
			6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	A-1 7.4 b Solve problems that can be modeled using pairs of linear equations in two variables, interpret the solutions and determine whether the solutions are reasonable.
			7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	A-2 7.2 a Solve systems of linear equations and inequalities in three variables by substitution and elimination.
			8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	A-2 7.2b Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.
			9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).	AERO needs to consider from three to two variables

Topic	Concept	Common Core	AERO
ALGEBRA	Reasoning With Equation and Inequalities	Represent and solve equations and inequalities graphically.	A-1 7.4 a Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.
		10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). 11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	A-1 7.4 b Solve problems that can be modeled using pairs of linear equations in two variables, interpret the solutions and determine whether the solutions are reasonable
		12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-plane	A-1 7.4c Graph with and without technology the solution set for a pair of linear inequalities in two variables. Use the graph to find the solution set

Topic	Concept	Common Core	AERO	
FUNCTIONS	Interpreting Functions	Understand the concept of a function and use function notation.	1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.	A-1 7.1a Find the zeros, domain and range of a function.
			2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	A-1 7.1b Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.
			3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.	A-2 7.7 a Write the recursive formula for arithmetic and geometric sequences and find specific terms of arithmetic and geometric sequences. A-2 7.7 b Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions. A-2 7.7 c Find partial sums of arithmetic and geometric series.

Topic	Concept	Common Core		AERO
FUNCTIONS	Interpreting Functions	Interpret functions that arise in applications in terms of the context.	4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	A-1 7.1c Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph and the factors of a polynomial expression
			5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	A-1 7.1b Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.
			6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	A-1 7.1b Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Topic	Concept	Common Core		AERO
FUNCTIONS	Interpreting Functions	<p>Analyze functions using different representations.</p>	<p>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p>	<p>A-1 7.6 a Graph quadratic functions.</p> <p>A-2 7.3 d Analyze, describe and sketch graphs of quadratic functions and include the lines of symmetry.</p> <p>A-2 7.3 e Determine how the graph of a parabola changes if a, b and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information.</p> <p>A-2 7.5 d Analyze, describe and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.</p>

			<p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>	<p>A-2 7.4 c Analyze, describe and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.</p> <p>A-2 7.5a Analyze, describe and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.</p> <p>A-2 7.6 a Analyze, describe and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.</p>
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Topic	Concept	Common Core		AERO
FUNCTIONS	Interpreting Functions	Analyze functions using different representations.	<p>8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</p>	<p>A-1</p> <p>7.6 b Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square and by using the quadratic formula</p> <p>A-2</p> <p>7.5 c Understand the properties of rational exponents and use the properties to simplify, multiply, divide and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.</p>
			<p>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum</p>	<p>Application of All of the indicators dealing with properties of functions</p>

Topic	Concept	Common Core		AERO
FUNCTIONS	Building Functions	Build a function that models a relationship between two quantities.	<p>1. Write a function that describes a relationship between two quantities.</p> <p>Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</p> <p>(+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</p>	These are applications of All of the indicators dealing with properties of functions
			2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	A-2 7.7 a Write the recursive formula for arithmetic and geometric sequences and find specific terms of arithmetic and geometric sequences

Topic	Concept	Common Core		AERO
FUNCTIONS	Building Functions	Build new functions from existing functions.	<p>3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs.</p> <p>Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p>	<p>A-2</p> <p>7.3 e Determine how the graph of a parabola changes if a, b and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information.</p>

Topic	Concept	Common Core	AERO	
FUNCTIONS	Building Functions	<p>Build new functions from existing functions.</p>	<p>4. Find inverse functions.</p> <p>Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.</p> <p>(+) Verify by composition that one function is the inverse of another.</p> <p>(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>(+) Produce an invertible function from a non-invertible function by restricting the domain.</p>	<p>Precalculus</p> <p>1.1f Find the composition of two functions and determine the domain and the range of the composite function . Conversely, when given a function, find two other functions for which the composition is the given one .</p> <p>Precalculus</p> <p>1.1g Define and find inverse functions, their domains and their ranges . Verify symbolically and graphically whether two given functions are inverses of each other</p>
			<p>5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.</p>	<p>A-2</p> <p>7.6 b Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.</p>

Topic	Concept	Common Core		AERO
FUNCTIONS	Linear, Quadratic, and Exponential Models	<p>Construct and compare linear, quadratic, and exponential models and solve problems.</p>	<p>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.</p> <p>Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</p> <p>Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p>	<p>Precalculus</p> <p>1.1a Use paper and pencil methods and technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise-defined functions . Use these graphs to solve problems, and translate among verbal, tabular, graphical and symbolic representations of functions by using technology as appropriate</p>
<p>2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p>				

Topic	Concept	Common Core		AERO
FUNCTIONS	Linear, Quadratic, and Exponential Models	Construct and compare linear, quadratic, and exponential models and solve problems.	3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	Precalculus 1.1a Use paper and pencil methods and technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise-defined functions . Use these graphs to solve problems, and translate among verbal, tabular, graphical and symbolic representations of functions by using technology as appropriate
			4. For exponential models, express as a logarithm the solution to $abct = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.	
		Interpret expressions for functions in terms of the situation they model.	5. Interpret the parameters in a linear or exponential function in terms of a context.	

Topic	Concept	Common Core		AERO
Function	Trigonometric Functions	Extend the domain of trigonometric functions using the unit circle.	1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Precalculus 3.1a Define and use the trigonometric ratios cotangent, secant and cosecant in terms of angles of right triangles .
			2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Precalculus 3.1b Model and solve problems involving triangles using trigonometric ratios . Precalculus 3.1c Develop and use the laws of sines and cosines to solve problems . Precalculus 3.1 d Develop and use radian measures of angles, measure angles in degrees and radians, and convert between degree and radian measures .
			3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x , $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.	Precalculus 3.1 e Deduce geometrically and use the value of the sine, cosine and tangent functions at 0 , $\pi/6$, $\pi/4$, $\pi/3$ and $\pi/2$ radians and their multiples .
			4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	Precalculus 3.2 a Analyze and graph trigonometric functions, including the translation of these trigonometric functions . Describe their characteristics (i.e., spread, amplitude, zeros, symmetry, phase, shift, vertical shift, frequency) .

Topic	Concept	Common Core		AERO
Functions	Trigonometric Functions	Model periodic phenomena with trigonometric functions.	5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	Precalculus 3.2 a Analyze and graph trigonometric functions, including the translation of these trigonometric functions . Describe their characteristics (i.e., spread, amplitude, zeros, symmetry, phase, shift, vertical shift, frequency) .
			6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	Precalculus 3.2 b Define, analyze and graph inverse trigonometric functions and find the values of inverse trigonometric functions .
			7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	
		Prove and apply trigonometric identities.	8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.	Precalculus 3.2 d Derive the fundamental Pythagorean trigonometric identities; sum and difference identities; half-angle and double-angle identities; and the secant, cosecant and cotangent functions . Use these identities to verify other identities and simplify trigonometric expressions .
			9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	

Topic	Concept	Common Core		AERO
GEOMETRY	Congruence	Experiment with transformations in the plane	1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	Geom 8.1b Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines by using appropriate geometric construction tools . Explain and justify the process used.
			2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	Geom 8.2 d Predict and describe the results of translations, reflections and rotations on polygons . Describe a motion or series of motions that will show that two shapes are congruent
			3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	
			4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments	

Topic	Concept	Common Core		AERO
GEOMETRY	Congruence	Experiment with transformations in the plane	5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another	Geom 8.2 b Identify types of symmetry (i.e., line, point, rotational, self-congruences) of polygons .
		Understand congruence in terms of rigid motions	6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	
			7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	Geom 8.2 c Solve problems involving congruent and similar polygons .
			8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions	Geom 8.2 m Use theorems to show if two triangles are congruent (i.e., SSS, SAS, ASA) or similar (i.e., AA, SAS, SSS) .

Topic	Concept	Common Core		AERO
GEOMETRY	Congruence	Prove geometric theorems	9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	Geom 8.1d Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines .
			10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180° ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	Geom Isosceles Triangles 8.2 p Prove and apply the isosceles triangle theorem and its converse . Geom Right Triangles 8.2 q Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane
			11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	Geom 8.2 f Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons . Understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle Geom 8.2 g Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity . Geom 8.2 h Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement .

Topic	Concept	Common Core		AERO
GEOMETRY	Congruence	Make geometric constructions	<p>12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.</p>	<p>Geom 8.3 a Construct the circle that passes through three given points not on a line . Construct tangents to circles . Circumscribe and inscribe circles . Justify the process used .</p> <p>Geom 8.1f Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane .</p>
			<p>13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.</p>	

Topic	Concept	Common Core		AERO
GEOMETRY	Similarity, Right Triangles, and Trigonometry	Understand similarity in terms of similarity transformations	<p>1. Verify experimentally the properties of dilations given by a center and a scale factor:</p> <p>A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</p>	<p>Geom 8.2 d Predict and describe the results of translations, reflections and rotations on polygons . Describe a motion or series of motions that will show that two shapes are congruent .</p>
			<p>2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides</p>	
			<p>3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar</p>	

Topic	Concept	Common Core		AERO
GEOMETRY	Similarity, Right Triangles, and Trigonometry	Prove theorems involving similarity	4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	Geom 8.2 q Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane .
			5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	Geom 8.2 h Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement .
		Define trigonometric ratios and solve problems involving right triangles	6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	Geom 8.2 q Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane .
			7. Explain and use the relationship between the sine and cosine of complementary angles.	Geom 8.2ss Define and use the trigonometric functions sine, cosine and tangent in terms of angles of right triangles .
			8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	Geom 8.2 u Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions . Interpret the solutions and determine whether the solutions are reasonable . Use technology as appropriate .

Topic	Concept	Common Core		AERO
GEOMETRY	Similarity, Right Triangles, and Trigonometry	Apply trigonometry to general triangles	9. (+) Derive the formula $A = 1/2 ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	<p>3.1a Define and use the trigonometric ratios cotangent, secant and cosecant in terms of angles of right triangles .</p> <p>3.1b Model and solve problems involving triangles using trigonometric ratios .</p> <p>.</p> <p>3.1 d Develop and use radian measures of angles, measure angles in degrees and radians, and convert between degree and radian measures .</p> <p>3.1 e Deduce geometrically and use the value of the sine, cosine and tangent functions at $0, \pi/6, \pi/4, \pi/3$ and $\pi/2$ radians and their multiples .</p>
			10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.	Precalculus 3.1c Develop and use the laws of sines and cosines to solve problems
			11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	

Topic	Concept	Common Core		AERO
GEOMETRY	Circles	Understand and apply theorems about circles	1. Prove that all circles are similar.	Geom 8.3 b Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant and tangent
			2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	
			3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	Geom 8.3 a Construct the circle that passes through three given points not on a line . Construct tangents to circles . Circumscribe and inscribe circles . Justify the process used .
			4. (+) Construct a tangent line from a point outside a given circle to the circle.	

Topic	Concept	Common Core		AERO
GEOMETRY	Circles	Find arc lengths and areas of sectors of circles	5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	Geom 8.3 c Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (i.e., central, inscribed and intersections of secants and tangents)
		Translate between the geometric description and the equation for a conic section	1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	Geom 8.3 e Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a, b and r change in the equation $(x - a)^2 + (y - b)^2 = r^2$.
GEOMETRY	Expressing Geometric properties With Equations		2. Derive the equation of a parabola given a focus and directrix.	A-2 7.3 e Determine how the graph of a parabola changes if a, b and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information.
			3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	No specific indicator.. recommend adding

Topic	Concept	Common Core		AERO
GEOMETRY	Expressing Geometric properties With Equations	Use coordinates to prove simple geometric theorems algebraically	4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.	Geom 8.1d Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines
			5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	
			6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	
			7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	

Topic	Concept	Common Core		AERO
GEOMETRY	Geometric Measurement and Dimension	<p>Explain volume formulas and use them to solve problems</p>	<p>1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.</p>	<p>Geom 8.3 d Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors .</p> <p>Geom 8.3 f Develop simple geometric proofs involving circles and provide reasons for each statement .</p> <p>Geom 8.4 c Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres . Relate these measures to each other using formulas .</p>
		<p>2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.</p>	<p>2. (+) Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.</p>	<p>Geom 8.4 c Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres . Relate these measures to each other using formulas .</p>
		<p>3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</p>	<p>3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.</p>	<p>Geom 8.4 a Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres</p>
		<p>Visualize relationships between two-dimensional and three-dimensional objects</p>	<p>4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.</p>	<p>Geom 8.4 d Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects .</p>

Topic	Concept	Common Core		AERO
GEOMETRY	Modeling With Geometry	Apply geometric concepts in modeling situations	1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	All applications of geometry indicators
			2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	
			3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on a single count or measurement variable	1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	A-1 7.7a Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.
			2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
			3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	
			4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Interpreting Categorical and Quantitative Data	Summarize, represent, and interpret data on two categorical and quantitative variables	<p>5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p>	<p>A-1</p> <p>7.7a Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.</p>
			<p>6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</p> <p>b. Informally assess the fit of a function by plotting and analyzing residuals.</p> <p>c. Fit a linear function for a scatter plot that suggests a linear association.</p>	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Interpreting Categorical and Quantitative Data	Interpret linear models	7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	A-1 7.7a Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.
			8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	
			9. Distinguish between correlation and causation	
Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Making Inferences and Justifying Conclusions	Understand and evaluate random processes underlying statistical experiments	1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	A-1 7.7b Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results.
			2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Making Inferences and Justifying Conclusions	Make inferences and justify conclusions from sample surveys, experiments, and observational studies	3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	A-1 7.7b Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results.
			4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	
			5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	
			6. Evaluate reports based on data.	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Conditional Probability and The Rules of Probability	Understand independence and conditional probability and use them to interpret data	1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	A-2 7.8 a Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.
			2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
			3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Conditional Probability and The Rules of Probability	Understand independence and conditional probability and use them to interpret data	<p>4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</p>	<p>A-2 7.8 a Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.</p> <p>7.8 b Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.</p>
			<p>5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer</p>	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Conditional Probability and The Rules of Probability	Use the rules of probability to compute probabilities of compound events in a uniform probability model	6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	A-2 7.8 a Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events. 7.8 b Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.
			7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	
			8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model.	
			9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Using Probability To Make Decisions	Calculate expected values and use them to solve problems	1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	<p>A-2 7.8 a Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.</p> <p>7.8 b Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.</p>
			2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	
			3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.	

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Using Probability To Make Decisions	Calculate expected values and use them to solve problems	4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?	<p>A-2 7.8 a Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.</p> <p>7.8 b Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.</p>

Topic	Concept	Common Core		AERO
STATISTICS AND PROBABILITY	Using Probability To Evaluate Decisions	Use probability to evaluate outcomes of decisions	<p>5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.</p> <p>a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.</p> <p>b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.</p>	<p>A-2</p> <p>7.8 a Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.</p> <p>7.8 b Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.</p>
			<p>6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).</p>	
			<p>7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).</p>	